GENERAL APTITUDE

Q. No. 1–5 Carry One Mark Each

1. As you grow older, an injury to your _________ may take longer to _________.

(A) heel / heel (B) heal / heel (C) heal / heal (D) heel / heal

2. In a 500 m race, P and Q have speeds in the ratio of 3 : 4. Q starts the race when P has already covered 140 m.

What is the distance between P and Q (in m) when P wins the race?

(A) 20 (B) 40 (C) 60 (D) 140

3. Three bells P, Q, and R are rung periodically in a school. P is rung every 20 minutes; Q is rung every 30 minutes and R is rung every 50 minutes.

If all the three bells are rung at 12:00 PM, when will the three bells ring together again the next time?

(A) 5:00 PM (B) 5:30 PM (C) 6:00 PM (D) 6:30 PM

4. Given below are two statements and four conclusions drawn based on the statements.

Statement 1: Some bottles are cups.

Statement 2: All cups are knives.

Conclusion I: Some bottles are knives.

Conclusion II: Some knives are cups.

Conclusion III: All cups are bottles.

Conclusion IV: All knives are cups.

Which one of the following options can be logically inferred?

(A) Only conclusion I and conclusion II are correct

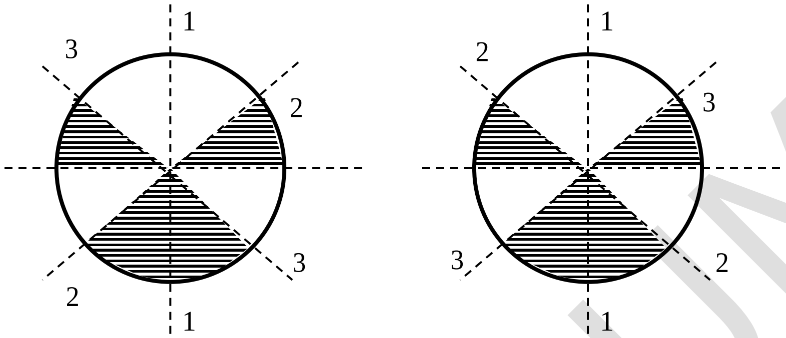
(B) Only conclusion II and conclusion III are correct

(C) Only conclusion II and conclusion IV are correct

(D) Only conclusion III and conclusion IV are correct

5. The figure below shows the front and rear view of a disc, which is shaded with identical patterns. The disc is flipped once with respect to any one of the fixed axes 1-1, 2-2 or 3-3 chosen uniformly at random.

What is the probability that the disc DOES NOT retain the same front and rear views after the flipping operation?



(A) 0 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) 1

Q. No. 6–10 Carry Two Marks Each

6. Altruism is the human concern for the wellbeing of others. Altruism has been shown to be motivated more by social bonding, familiarity and identification of belongingness to a group. The notion that altruism may be attributed to empathy or guilt has now been rejected.

Which one of the following is the CORRECT logical inference based on the information in the above passage?

(A) Humans engage in altruism due to guilt but not empathy

(B) Humans engage in altruism due to empathy but not guilt

(C) Humans engage in altruism due to group identification but not empathy

(D) Humans engage in altruism due to empathy but not familiarity

7. There are two identical dice with a single letter on each of the faces. The following six letters: Q, R, S, T, U, and V, one on each of the faces. Any of the six outcomes are equally likely. The two dice are thrown once independently at random. What is the probability that the outcomes on the dice were composed only of any combination of the following possible outcomes: Q, U and V?

(A) $\frac{1}{4}$ (B) $\frac{3}{4}$ (C) $\frac{1}{6}$ (D) $\frac{5}{36}$

8. The price of an item is 10% cheaper in an online store S compared to the price at another online store M. Store S charges ₹ 150 for delivery. There are no delivery charges for orders from the store M. A person bought the item from the store S and saved ₹ 100.

What is the price of the item at the online store S (in ₹) if there are no other charges than what is described above?

(A) 2500 (B) 2250 (C) 1750 (D) 1500

9. The letters P, Q, R, S, T and U are to be placed one per vertex on a regular convex hexagon, but not necessarily in the same order.

Consider the following statements:

- The line segment joining R and S is longer than the line segment joining P and Q.
- The line segment joining R and S is perpendicular to the line segment joining P and Q.
- The line segment joining R and U is parallel to the line segment joining T and Q.

Based on the above statements, which one of the following options is CORRECT?

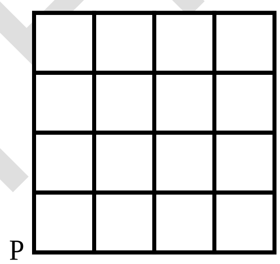
(A) The line segment joining R and T is parallel to the line segment joining Q and S

(B) The line segment joining T and Q is parallel to the line joining P and U

(C) The line segment joining R and P is perpendicular to the line segment joining U and Q

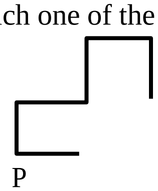
(D) The line segment joining Q and S is perpendicular to the line segment joining R and P

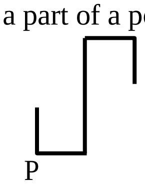
10.

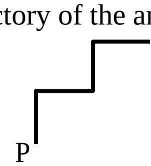


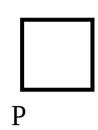
An ant is at the bottom-left corner of a grid (point P) as shown above. It aims to move to the top-right corner of the grid. The ant moves only along the lines marked in the grid such that the current distance to the top-right corner strictly decreases.

Which one of the following is a part of a possible trajectory of the ant during the movement?

(A)


(B)


(C)


(D)


ELECTRICAL ENGINEERING

Q. No. 11-35 Carry One Mark Each

11. The transfer function of a real system, (s), is given as:

$$H(s)=\frac{As+B}{s^2+CS+D},$$

where A, B, C and D are positive constants. This system cannot operate as

(A) low pass filt**er.**
(B) high pass filter.
(C) band pass filt**er.**
(D) an integrator.

Key: (A)

Sol: → We can know the filter characteristic by evaluating the transfer function magnitude at 2 different value of ω i.e., $\omega=0$ and $\omega=\infty$ $(s=0, s=\infty)$

$$H(s)=\frac{As+B}{s^2+Cs+D}$$

$$H(s)=\frac{As+B}{s^2+CS+D}$$

→ $H(0)=\frac{B}{D}$ (i.e., low frequency are passed)

$H(\infty)=0$ (i.e., high frequency are blocked)

Hence it represents a low pass filter.

12. For an ideal MOSFET biased in saturation, the magnitude of the small signal current gain for a common drain amplifier is

(A) 0
(B) 1
(C) 100
(D) infinite

Key: (D)

Sol: For common drain MOSFE based amplifier is having current gain as

$$A_I=\frac{I_s}{I_g}$$

Where: I_s = source current

I_g = gate current

As we know that for ideal MOSFET gate current in almost zero (i.e., $I_g \simeq 0$)

So, $\boxed{A_I=\frac{I_s}{0}=\infty}$

13. The most commonly used relay, for the protection of an alternator against loss of excitation, is

(A) offset Mho relay. (B) over current relay.

(C) differential relay. (D) Buchholz relay.

Key: (A)

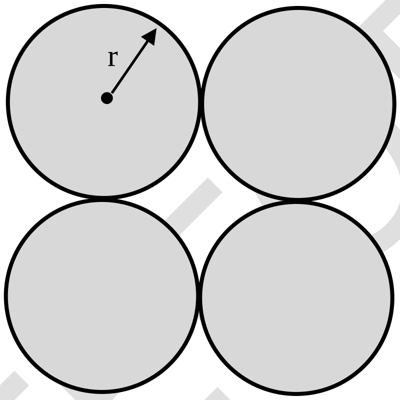
Offset mho relays is used for fast-detection of the loss of excitation in the alternator based on impedance.

(B) Over current relay sense over current in the system.

(C) Differential relays are used for internal fault between protected winding by differential protection.

(D) Buchholz relay is used for transformer protection.

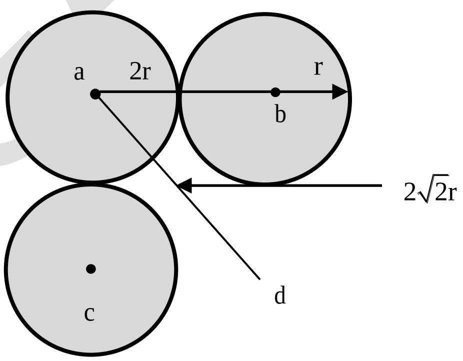
14. The geometric mean radius of a conductor, having four equal strands with each strand of radius 'r', as shown in the figure below, is



(A) 4r (B) 1.414r (C) 2r (D) 1.723r

Key: (D)

Given each, conductor radius = r



$\text{GMR} = \left[D_{bb}\ \ D_{ab}\ \ D_{bc}\ \ D_{bd}\right]^{1/4} = \left[0.7888r.2r \times 2\sqrt{2r} \times 2r\right]^{1/4}$

$\therefore \boxed{\text{GMR} = 1.723r}$

Hence option (C) is the correct option.

15. The valid positive, negative and zero sequence impedances (in p.u.), respectively, for a 220 kV, fully transposed three-phase transmission line, from the given choices are

(A) 1.1, 0.15 and 0.08
(B) 0.15, 0.15 and 0.35
(C) 0.2, 0.2 and 0.2
(D) 0.1, 0.3 and 0.1

Key: (B)

For $3-\phi$ fully transposed transmission line.

Relationship between X_1, X_2 & X_0 is $\boxed{X_1 = X_2 < X_0}$

X_0 is higher due to return line

Now, to check the correct option we need to verify above condition

(A) $1.1 \neq 0.15 > 0.08$ Hence invalid

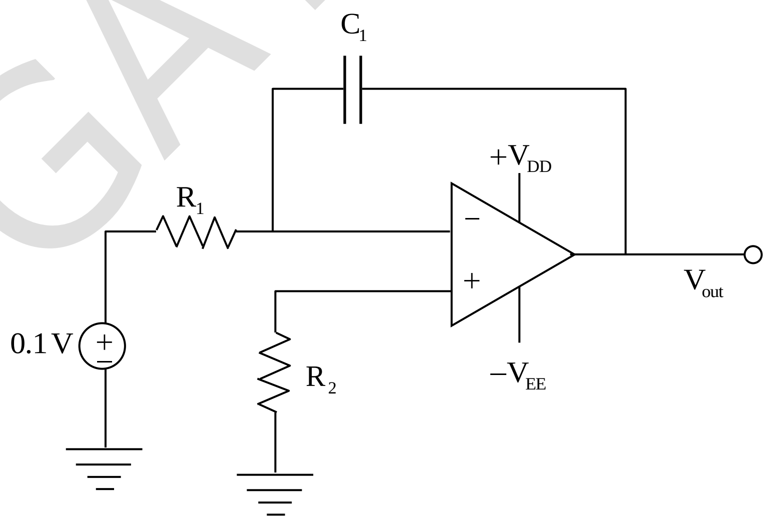
(B) 0.15 = 0.15 < 0.35 Hence valid

(C) 0.2 = 0.2 = 0.2 Hence invalid

(D) 0.1 $\neq$ 0.3 $\neq$ 0.1 Hence invalid

$\therefore$ From above discussion we can see (B) is valid solution.

16. The steady state output ((V_{out}), of the circuit shown below, will

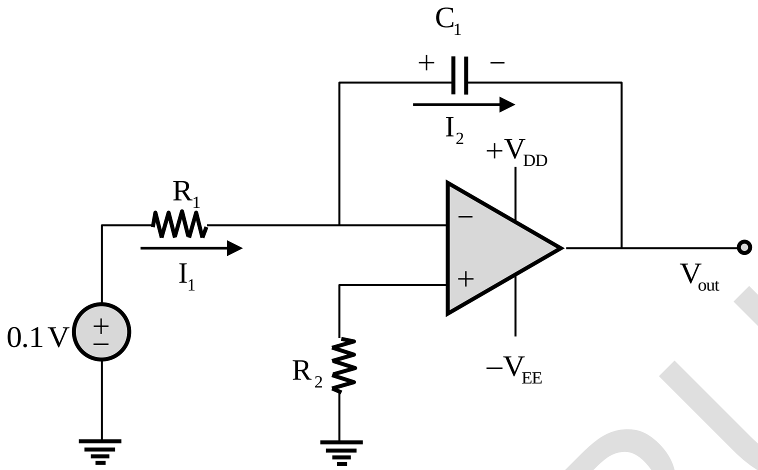


(A) saturate to $+V_{DD}$ (B) saturate to $-V_{EE}$

(C) become equal to 0.1 V (D) become equal to –0.1 V

Key: (B)

Sol:



$$I_1 = I_2$$

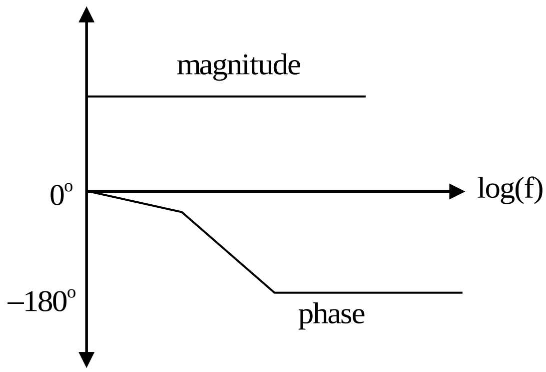
$$\frac{0.1-0}{R_1} = -C_1\frac{dV_o}{dt}$$

$$\frac{dV_o}{dt} = \frac{-0.1}{R_1C_1}$$

$$V_o = \frac{-0.1}{R_1C_1}\int dt = -0.1\left(\frac{t}{R_1C_1}\right)$$

In steady at $t-\infty$; So $V_o \to -\infty$ it means op-map will saturated at $-V_{EE}$.

17. The Bode magnitude plot of a first order stable system is constant with frequency. The asymptotic value of the high frequency phase, for the system, is $-180°$. This system has



(A) one LHP pole and one RHP zero at the same frequency.

(B) one LHP pole and one LHP zero at the same frequency.

(C) two LHP poles and one RHP zero.

(D) two RHP poles and one LHP zero.

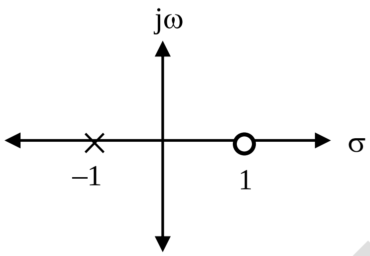
Key: **(C, D)**

Sol: → Since it is given that order of system is 1 it means it has only 1 pole.

(Option C, D are eliminated)

→ Since the magnitude plot is constant for all frequency it means the system is like an all pass filter. Hence there must be an zero such that the pole-zero plot should be symmetrical w.r.t $j\omega$ axis.

→ Example: $k\left(\frac{s-1}{s+1}\right) \Rightarrow$



18. A balanced Wheatstone bridge $ABCD$ has the following arm resistances:

$R_{AB} = 1k\Omega \pm 2.1\%, R_{BC} = 100\Omega \pm 0.5\%$; R_{CD} is an unknown resistance;

$R_{DA} = 300\Omega \pm 0.4\%$. The value of R_{CD} and its accuracy is

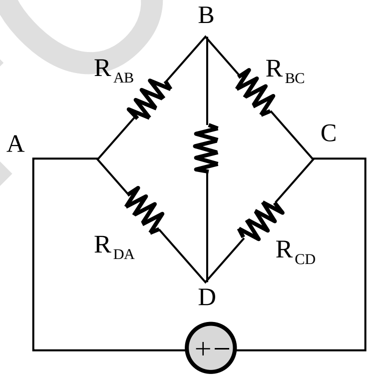
(A) $30\Omega \pm 3\Omega$ (B) $30\,\Omega \pm 0.9\,\Omega$ (C) $3000\,\Omega \pm 90\,\Omega$ (D) $3000\,\Omega \pm 3\,\Omega$

Key: **(B)**

Sol: As per the given data

Since it is given that bridge is balanced

We can say



$R_{AB}.R_{CD} = R_{BC}.R_{DA}$

$R_{CD} = \frac{R_{BC}.R_{DA}}{R_{AB}} = \frac{(100 \pm 0.5\%)(300 \pm 0.4)}{(1000 + 2.1\%)}$

$= \left(\frac{100 \times 300}{1000}\right) \pm (0.5 + 0.4 + 2.1)\%$

$= 30 \pm 3\%$

$= 30 \pm \left(30 \times \frac{3}{100}\right)$

$= (30 \pm 0.9)\Omega$

19. The open loop transfer function of a unity gain negative feedback system is given by $G(s) = \frac{k}{s^2 + 4s - 5}$.

The range of *k* for which the system is stable, is

(A) $k > 3$ (B) $k < 3$ (C) $k > 5$ (D) $k < 5$

Key: **(C)**

Sol: $\rightarrow$ The characteristic equation of the given system is

$$1+G(s)=0$$

$$\Rightarrow 1+\frac{k}{s^2+45s-5}=0$$

$$\Rightarrow s^2+4s-5+k=0$$

For stability all co-efficient of the above equation should be of same sign, so $(k-5)>0$ or $k>5$.

20. Consider a 3 × 3 matrix A whose (i,j)-th element, $a_{i,j}=(i-j)^3$. Then the matrix A

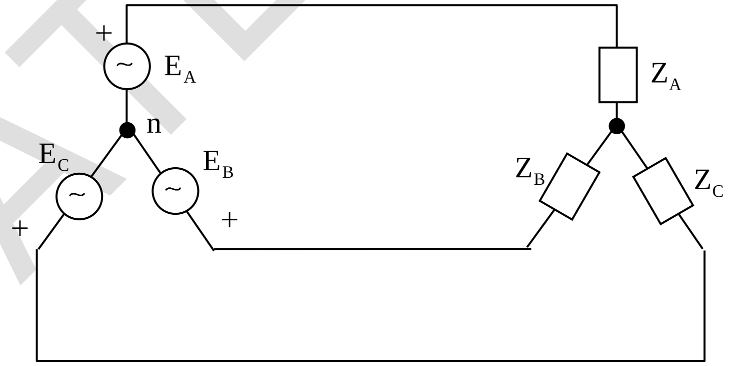
(A) symmetric. (B) skew-symmetric. (C) unitary (D) null.

Key: **(B)**

Sol: Let $A_{3\times3}=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, where $a_{ij}=(i-j)^3$ then $A_{3\times3}=\begin{bmatrix} 0 & -1 & -8 \\ 1 & 0 & -1 \\ 8 & 1 & 0 \end{bmatrix} \Rightarrow A^T=-A$

$\therefore$ A is skew-symmetric

21. In the circuit shown below, a three-phase star-connected unbalanced load is connected to a balanced three-phase supply of $100\sqrt{3}V$ with phase sequence ABC. The star connected load has $Z_A=10\,\Omega$ and $Z_B=20\angle 60^o\,\Omega$.. The value of Z_C in Ω , for which the voltage difference across the nodes n and n' is zero, is



(A) $20\angle -30^o$ (B) $20\angle 30^o$ (C) $20\angle -60^o$ (D) $20\angle 60^o$

Key: **(C)**

Sol: $\rightarrow$ Since the source is balanced $V_n=0$,

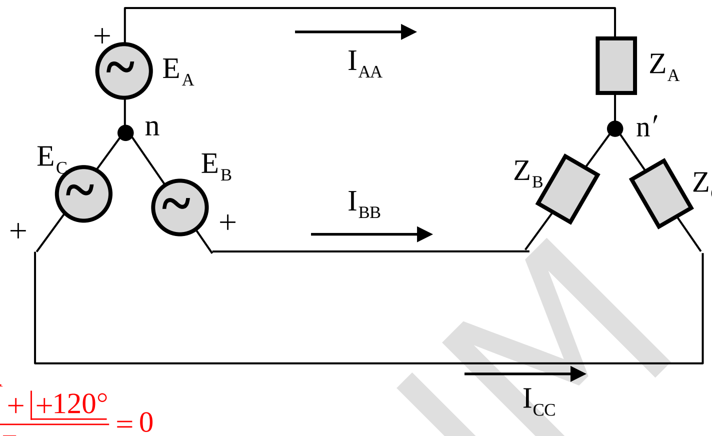
We need $V_n - V_{n1}=0$ it means $V_{n1}=0$

→ Since the sequence ABC we can say

$E_A = 100\sqrt{3}\underline{|0°}$

$E_B = 100\sqrt{3}\underline{|-120°}$

$E_C = 100\sqrt{3}\underline{|120°}$



By KCL at node n we can write

$I_{AA} + I_{BB} + I_{CC} = 0$

$\Rightarrow \frac{E_A}{Z_A} + \frac{E_B}{Z_B} + \frac{E_C}{Z_C} = 0$

$\Rightarrow \frac{100\sqrt{3}\underline{|0°}}{10} + \frac{100\sqrt{3}\underline{|-120°}}{20\underline{|60°}} + \frac{100\sqrt{3} + \underline{|+120°}}{Z_C} = 0$

$\Rightarrow 10\sqrt{3} + \left(5\sqrt{3}\underline{|-180°}\right) = -\frac{100\sqrt{3}\underline{|120°}}{Z_C}$

$\Rightarrow 10\sqrt{3} - 5\sqrt{3} = -\frac{100\sqrt{3}\underline{|120°}}{Z_C}$

$\Rightarrow \quad Z_C = \frac{-100\sqrt{3}\underline{|120°}}{5\sqrt{3}} = -20\underline{|120°} = (-1)(20\underline{|120°}) = (1\underline{|-180°})(20\underline{|120°}) = 20\underline{|-60°}\ \Omega$

22. A charger supplies 100 W at 20 V for charging the battery of a laptop. The power devices, used in the converter inside the charger, operate at a switching frequency of 200 kHz. Which power device is best suited for this purpose?

(A) IGBT (B) Thyristor

(C) MOSFET (D) BJT

Key: (C)

Sol: Given, switching frequency = 200 kHz, which is high enough for high switching frequency MOSFET power devices are most suitable. When as thyristor is preferred for low switching frequency high power handling.

23. A long conducting cylinder having a radius ‘b’ is placed along the z axis. The current density is $J = J_a r^3 \hat{z}$ for the region r < b where r is the distance in the radial direction. The magnetic field intensity (**H**) for the region inside the conductor (i.e. for r < b) is

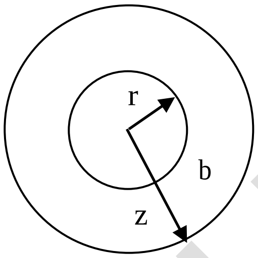
(A) $\frac{J_a}{4} r^4$ (B) $\frac{J_a}{3} r^3$ (C) $\frac{J_a}{5} r^4$ (D) $J_a r^3$

Key: (C)

Given,

$\vec{J} = J_a r^2 \hat{z}$ for region $r < b$.

We need to calculate magnetic field intensity $(\vec{H})$ ro $r < 6$



$$I = \int_s \vec{J}.d\vec{s}, \qquad \text{where } d\vec{s} = \text{rad } d\phi\hat{z}$$

$$\Rightarrow \quad I = \int J_a r^3 \hat{z}.\, rdr\, d\phi\, \hat{z}$$

$$= J_a \int_{r=0}^{r} r^4 dr \int_{\phi=0}^{2\pi} d\phi$$

$$= J_a \frac{r^5}{5}\Big|_b^r . \phi\Big|_0^{2\pi} = \frac{J_a(2\pi)r^5}{5}$$

$$\therefore \quad \text{As } \oint \vec{H}.\overrightarrow{dL} = I_a = \int_s \vec{J}.r\vec{s}$$

$$H(2\pi r) = \frac{J_a(2\pi)r^5}{5}$$

$$\therefore \boxed{H = \frac{J_a r^4}{5}}$$

Hence the option is (C).

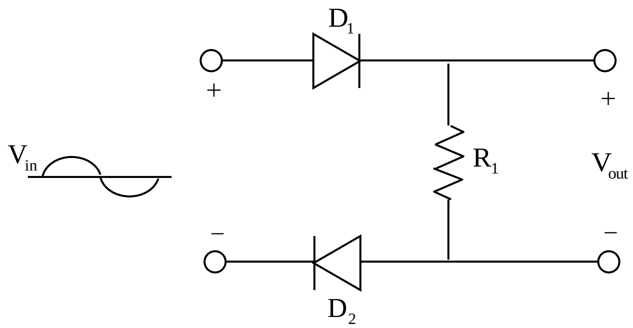
24. The type of single-phase induction motor, expected to have the maximum power factor during steady state running condition, is

(A) split phase (resistance start).
(B) shaded pole.
(C) capacitor start.
(D) capacitor start, capacitor run.

Key: (D)

Sol: The capacitor start, capacitor run type of single-phase induction motor, expected to have the maximum power factor during steady state running, condition. Since C in running & start time will neutralize the inductance effect-due to its winding. Hence power factor will be get improved and it will be better than split phase, shaded pole and capacitor start.

25. For the circuit shown below with ideal diodes, the output will be



(A) $V_{out} = V_{in}$ for $V_{in} > 0$

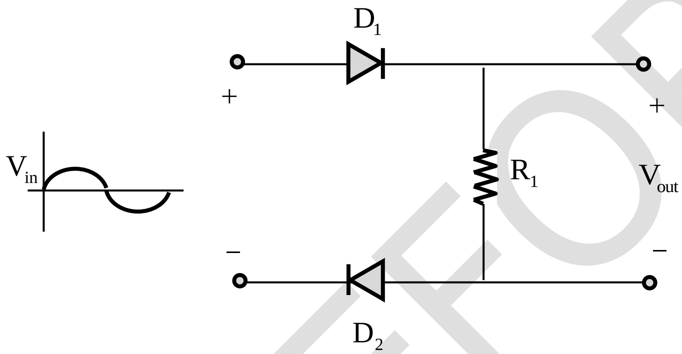
(B) $V_{out} = V_{in}$ for $V_{in} < 0$

(C) $V_{out} = -V_{in}$ for $V_{in} > 0$

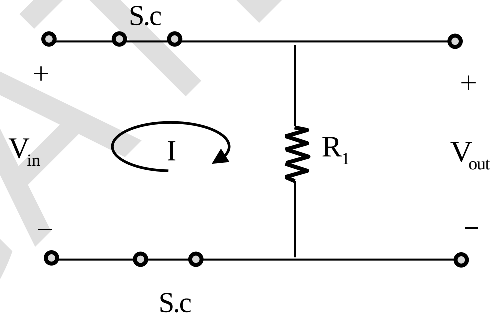
(D) $V_{out} = -V_{in}$ for $V_{in} < 0$

Key: (A)

Sol:



For +ve half cycle, both the diodes will be "ON' and these are ideal diodes, so it will act as short circuit.



$\boxed{V_{out} = V_{in}}$ for $V_{in} > 0V$

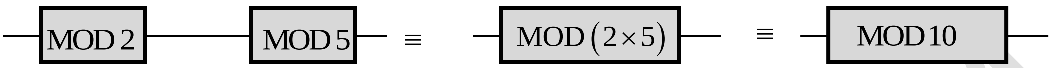
For –ve half cycle, both the diodes will be "OFF" and acts as open circuited.

$\boxed{V_{out} = 0V}$ for $V_{in} < 0V$

26. A MOD 2 and a MOD 5 up-counter when cascaded together results in a MOD______counter. (in integer)

Key: **(10)**

Sol: When counters of various mod values are cascaded then the value of overall modulus is product of individual mod.



→ So equivalently it's a MOD 10 counter.

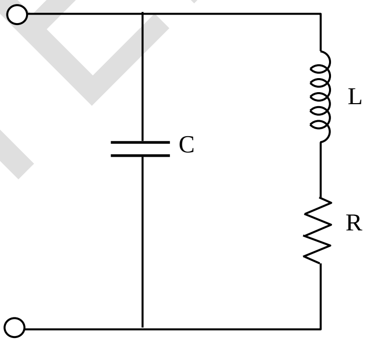
27. An inductor having a Q-factor of 60 is connected in series with a capacitor having a Qfactor of 240. The overall Q-factor of the circuit is ________. (round off to nearest integer)

Key: **(48)**

Sol: Whenever non ideal inductor having Q factor (Q_L) is connected in series with non ideal capacitor having Q factor (Q_C) then the overall Quality factor Q_T is given by

$$Q_T = \frac{Q_L Q_C}{Q_L + Q_C} = \frac{60 \times 240}{60 + 240} = 48$$

28. The network shown below has a resonant frequency of 150 kHz and a bandwidth of 600 Hz. The Q-factor of the network is __________. (round off to nearest integer)



Key: **(250)**

Sol: In general Bandwidth $= \frac{f_0}{Q}$

$$\Rightarrow \quad Q = \frac{f_0}{\text{Bandwidth}} = \frac{150\text{ kHz}}{600\text{ Hz}} = \frac{150 \times 10^3}{600} = \frac{1000}{4} = 250$$

29. The maximum clock frequency in MHz of a 4-stage ripple counter, utilizing flip-flops, with each flip-flop having a propagation delay of 20 ns, is ___________.(round off to one decimal place)

Key: **(12.5)**

Sol: For n stage ripple counter, we have the condition

$$T_{clk} \geq n\,t_{pd} \Rightarrow f_{clk} \leq \frac{1}{nt_{pd}}$$

$$\left(f_{clk}\right)_{max} = \frac{1}{nt_{pd}} = \frac{1}{4\times 20\times 10^{-9}} = \frac{1000}{80}\times 10^{6}\ \text{Hz} = 12.5\,\text{MHz}$$

30. If only 5% of the supplied power to a cable reaches the output terminal, the power loss in the cable, in decibels, is _________. (round off to nearest integer)

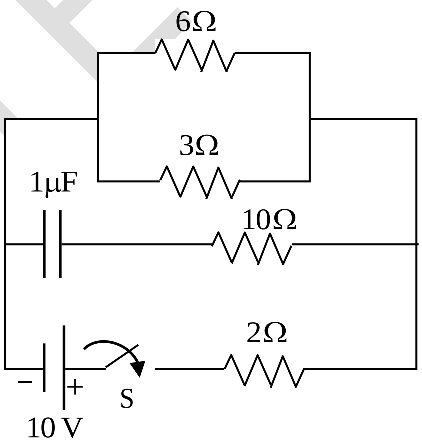
Key: **(13)**

Sol: Since% power at out terminal = 5%

$\therefore$ Power loss = 95%

We need to express the power loss in decibels $= 10\log\left(\frac{95}{5}\right) = 12.78 \simeq 13\,\text{dB}$

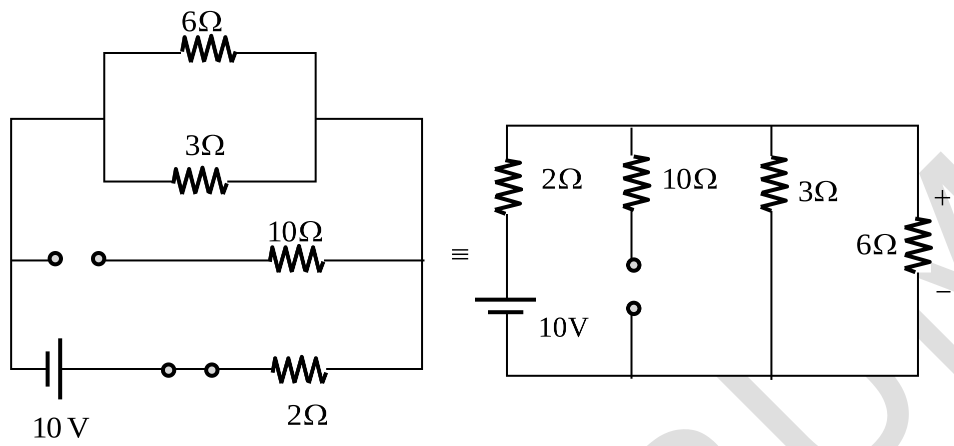
31. In the circuit shown below, the switch S is closed at t = 0. The magnitude of the steady state voltage, in volts, across the 6 Ω resistor is _________. (round off to two decimal places).



Key: **(5)**

Sol: At steady state $(t=\infty)$, switch is closed and capacitor is replaced by open circuit.

So the circuit at $t=\infty$ becomes



$$V_x = \frac{(6\|3)}{2+(6\|3)}10V = \frac{2\times10}{2+2} = 5V$$

32. A single-phase full-bridge diode rectifier feeds a resistive load of 50 Ω from a 200 V, 50 Hz single phase AC supply. If the diodes are ideal, then the active power, in watts, drawn by the load is _____________. (round off to nearest integer).

Key: **(800)**

Sol: Given, $1-\phi$ full bridge, with R load, $R_L = 5\,\Omega$

$V_{AC} = 200V, 50\text{ Hz}.$ The we need to calculate the active power drown by the load.

Since load is purely R.

$$\therefore \quad P_{o\,avg} = \frac{V_{o\,rms}^2}{f} = \frac{(200)^2}{50}$$

$$\boxed{V_{o\,rms} = \frac{V_m}{\sqrt{2}} = V_s}$$

$$= \frac{200\times200}{50} = 800\text{ W}$$

Hence active power drawn by the load is 800 W.

33. The voltage at the input of an AC-DC rectifier is given by $v(t)=230\sqrt{2}\sin\omega t$ where $\omega = 2\pi\times50\text{ rad/s}.$ rad/s. The input current drawn by the rectifier is given by

$$i(t) = 10\sin\left(\omega t - \frac{\pi}{3}\right) + 4\sin\left(3\omega t - \frac{\pi}{6}\right) + 3\sin\left(5\omega t - \frac{\pi}{3}\right).$$

The input power factor, (rounded off to two decimal places), is, ________________ lag.

Key: **(0.4472)**

Sol: $\rightarrow$ One of the way to calculate power factor is

$$\cos\phi = \frac{P}{|S|}$$

$\rightarrow$ In the given case only fundamental component will contribute in average power P.

$$P = V_{rms} I_{rms} \cos(\theta_V . \theta_I) = \left[\frac{230\sqrt{2}}{\sqrt{2}}\right]\left[\frac{10}{\sqrt{2}}\right]\cos\left[0-\left(-\frac{\pi}{3}\right)\right] = \frac{2300\sqrt{2}}{2}[\cos 60°]$$

$$= \frac{2300\sqrt{2}}{4} = 813.17 \text{ watt}$$

$\rightarrow$ $|S| = |V_{rms}||I_{rms}|$

$$= \left[\frac{230\sqrt{2}}{\sqrt{2}}\right]\sqrt{\left(\frac{10}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2\left(\frac{3}{\sqrt{2}}\right)^2} = [230]\sqrt{50+8+4.5} = 230\sqrt{62.5}$$

$$= 1818.30 \text{ V.A}$$

$\rightarrow$ $\text{Power factor} = \frac{P}{|S|} = \frac{813.17}{1818.30} = 0.4472$

Or

Given,

$V_{AC}(t) = 230\sqrt{2}\sin\omega t$ (For AC to DC rectifier i / p), $\omega = 100\,\pi$ rad/sec

$$i_{in}(t) = 10\sin\left(\omega t - \frac{\pi}{3}\right) + 4\sin\left(3\omega t - \frac{\pi}{6}\right) + 3\sin\left(5\omega t - \frac{\pi}{3}\right)$$

Then we need to calculate input power factor.

Input pf. $= g \times FDF$

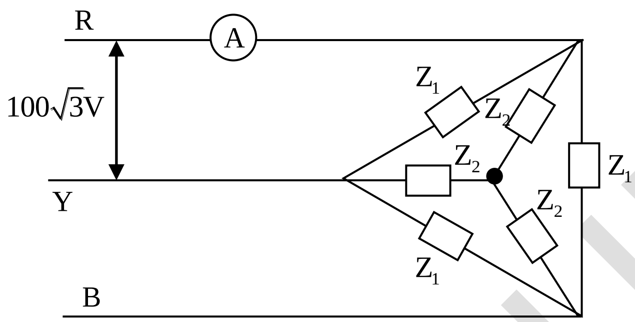
$$= \frac{I_{s1}}{I_{sr}}.\cos(\alpha_1), \quad \alpha_1 = \frac{\pi}{3}$$

$$= \frac{10/\sqrt{2}}{\sqrt{\left(\frac{10}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2 + \left(\frac{3}{\sqrt{2}}\right)^2}} \times \cos\frac{\pi}{3}$$

$$= 0.447$$

$\therefore$ The input power factor (rounded off to two decimal places) is 0.45.

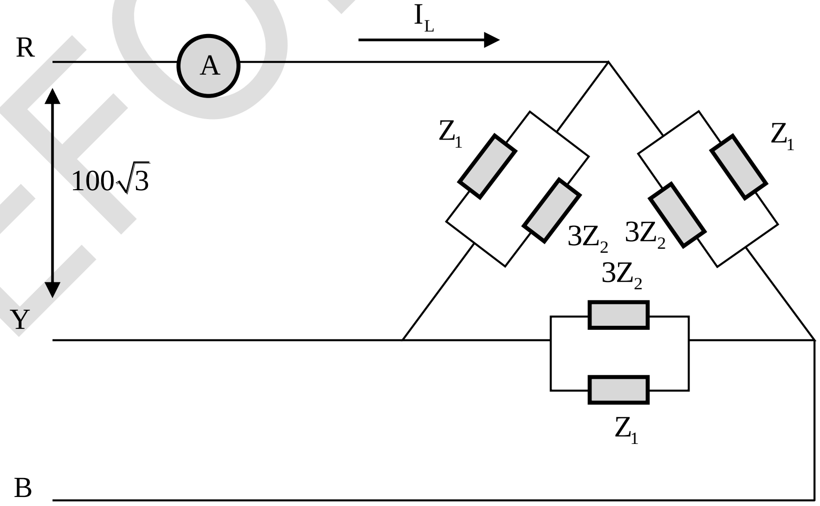
34. Two balanced three-phase loads, as shown in the figure, are connected to a $100\sqrt{3}V$, three-phase, 50 Hz main supply. Given $Z_1 = (18 + j24)\Omega$ and $Z_2 = (6 + j8)\Omega$. The ammeter reading, in amperes, is ________. (round off to nearest integer)



Key: (20)

Sol: If we convert the inner star network into its delta equivalent then each arm of delta will contain impedance having value $3Z_2$ ($\because$ All 3 impedance of star are same)

$$\rightarrow Z_{eq} = Z_1 \| 3Z_2 = \frac{3Z_1Z_2}{Z_1 + 3Z_2} = \frac{3(18+j24)(6+j8)}{(18+j24)+3(6+j8)} = \frac{(18+j24)(18+j24)}{(18+j24)(18+j24)} = \frac{18+j24}{2} = (9+j12)\Omega$$



$\rightarrow$ In the given circuit we have delta connected load and we need to obtain ammeter reading i.e., I_L

$$\text{In } \Delta \text{ load: } I_L = \sqrt{3}I_{ph} = \sqrt{3}\frac{V_{ph}}{Z_{eq}} = \sqrt{3}\frac{100\sqrt{3}}{9+j12} = \frac{300}{9+j12}$$

$$\rightarrow |I_L| = \frac{300}{\sqrt{9^2+12^2}} = 20A \text{ (ammeter read magnitude)}$$

35. The frequencies of the stator and rotor currents flowing in a three-phase 8-pole induction motor are 40 Hz and 1 Hz, respectively. The motor speed, in rpm, is ________________. (round off to nearest integer)

Key: (585)

Sol: Given, $f_s = 40H$, Rotor frequency $(f_r) = 1\,Hz$ and Number of poles (P) = 8

$\because \quad f_r = sfs \qquad \therefore s = \frac{1}{40} = 0.025$

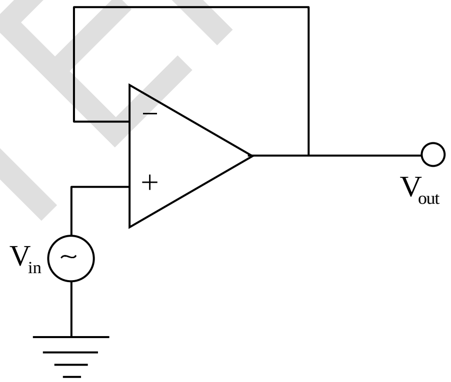
$$N_s = \frac{120 \times 40}{8} = 600 \text{ rpm}$$

$\therefore \quad N_r\,(\text{Motor speed}) = 600\{1 - 0.025\} = 585 \text{ rpm}$

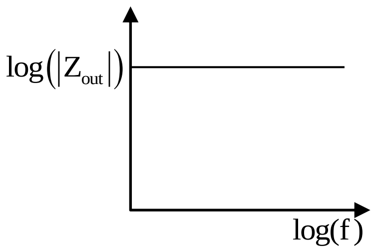
Hence, the motor spaced, in rpm is 585 rpm.

Q. No. 36–65 Carry Two Marks Each

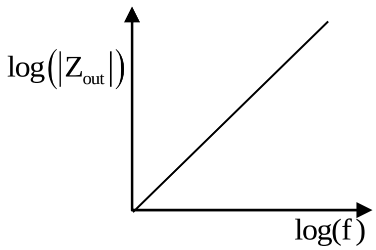
36. The output impedance of a non-ideal operational amplifier is denoted by Z_{out}. The variation in the magnitude of Z_{out} with increasing frequency, f, in the circuit shown below, is best represented by



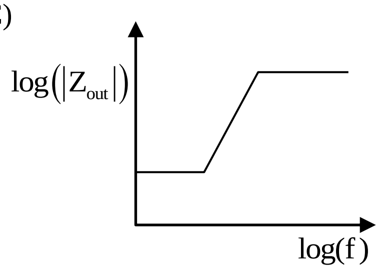
(A)



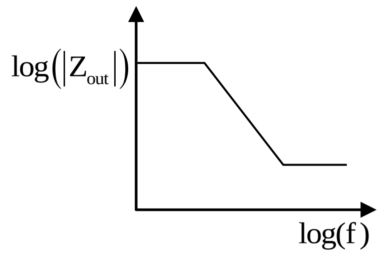
(B)



(C)



(D)



Key: (C)

Sol: Given circuit is voltage-series feedback amplifier with feedback factor $\beta = 1$

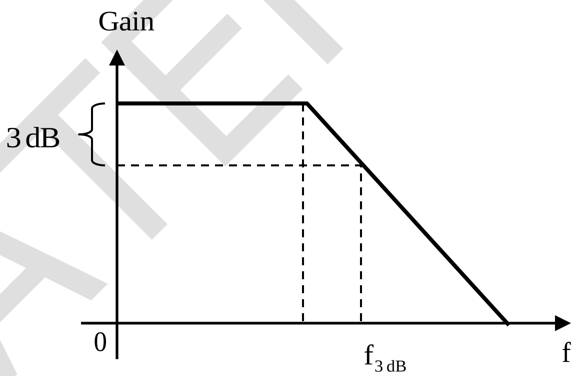
$$Z_{ot} = \frac{Z_{out}}{1 + A\beta}; \text{ where } Z_{of} \to \text{output impedance with feedback}$$

$$Z_{of} = \frac{Z_{out}}{1 + A}; \qquad (\because \beta = 1)$$

The gain of the amplifier with frequency is given by

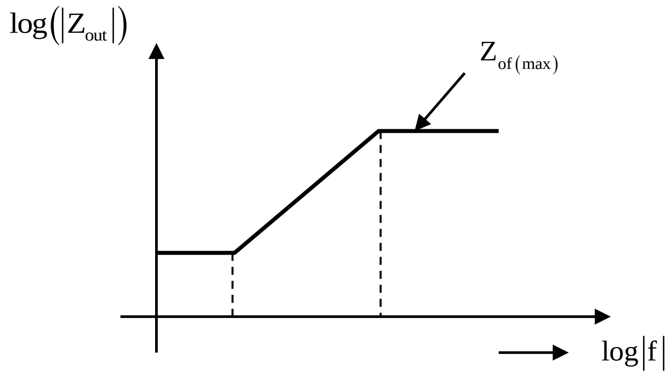
$$A = \frac{A_{OL}}{\sqrt{1 + \frac{f}{f_c}}}$$

From the equation as f increases, A decrease and Z_{of} increases.



When f is constant Z_{of} is constant. When $f \to \infty$, $A \to 0$, $Z_{of} \to Z_{of(max)}$

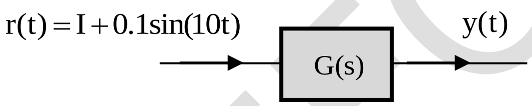
Combining all these points, we can draw the graph as



37. An LTI system is shown in the figure where

$$G(s)=\frac{100}{s^2+0.1s+10}.$$

The steady state output of the system, to the input (t), is given as $y(t)=a+b\sin b(10t+\theta)$. The values of 'a' and 'b' will be



(A) a = 1,b = 10 (B) a = 10, b = 1

(C) a = 1, b = 100 (D) a = 100, b = 1

Key: (B)

Sol: → Since the input is d.c with sinusoid, output will also be d.c with sinusoid (with same frequency as input sinusoid).

→ In general if

$$r(t)=1+0.1\sin 10t$$

$$y(t)=\left[\left|G(0)\right|.1\right]+\left[\left|G(10)\right|0.1\sin\left(10t+G(10)\right)\right]$$

→ $$G(s)=\frac{100}{s^2+0.1s+10}$$

$$G(0)=\frac{100}{10}=10$$

→ $$G(j\omega)=\frac{100}{(j\omega)^2+0.1j\omega+10}=\frac{100}{\left(10-\omega^2\right)+j\left(0.1\omega\right)}$$

$$= \frac{100}{\sqrt{(10-\omega^2)^2 + (0.1\omega)^2}} \angle -\tan^{-1}\left(\frac{0.1\omega}{10-\omega^2}\right)$$

$$\rightarrow \quad G(j10) = \frac{100}{\sqrt{(10-10^2)^2 + (0.1\times 10)^2}} \angle -\tan^{-1}\left(\frac{0.1\times 10}{10-10^2}\right)$$

$$= \frac{100}{\sqrt{(-90)^2 + 1^2}} \angle -\tan^{-1}\left(\frac{1}{-90}\right)$$

$$= \frac{100}{90.0055} \angle +\tan^{-1}\left(\frac{1}{90}\right)$$

$$= 1.11 \angle 0.63^\circ$$

$$\rightarrow \quad y(t) = (10.1) + \left[1.11\times 0.1\sin(10t + 0.63^\circ)\right]$$

$$= 10 + 0.11\sin(10t + 0.63^\circ)$$

$$= a + b\sin(10t + \theta)$$

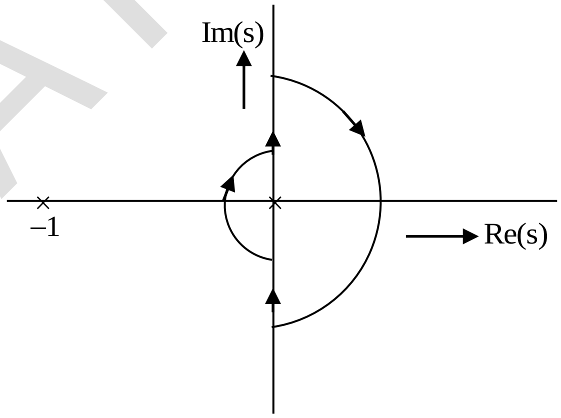
By comparison a = 10, b = 0.11, $\theta = 0.63^\circ$

→ Based on this no option correct however w.r.t value of a option B is closest answer.

38. The open loop transfer function of a unity gain negative feedback system is given as

$$G(s) = \frac{1}{s(s+1)}.$$

The Nyquist contour in the *s*-plane encloses the entire right half plane and a small neighbourhood around the origin in the left half plane, as shown in the figure below. The number of encirclements of the point $(-1 + j0)$ by the Nyquist plot of (s), corresponding to the Nyquist contour, is denoted as N. Then N equals to



(A) 0 (B) 1 (C) 2 (D) 3

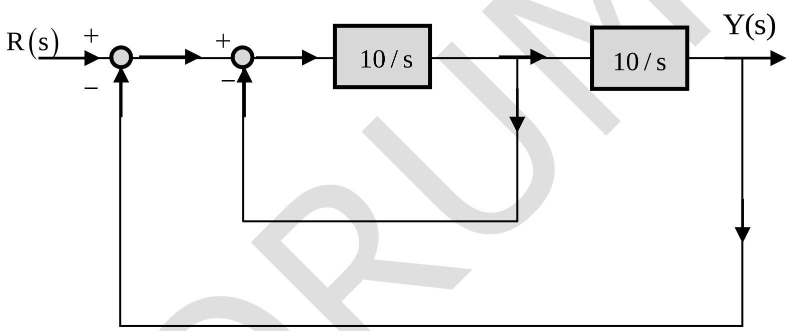
Key: (B)

Sol: → P = Number of poles encircled by the Nyquist contour, here P = 1(Pole at origin).

→ The characteristic equation of the system is $1+G(s)=0$ i.e., $s^2+s+1=0$, which represent a stable system (∵ all co-efficient of Characteristic equation are positive)

→ The equivalent Nyquist stability criterion is N = P since P = 1, we can say N = 1.

39. The damping ratio and undamped natural frequency of a closed loop system as shown in the figure, are denoted as ζ and ω, respectively. The values of ζ and ω are



(A) $\zeta=0.5$ and $\omega_n=10\ \text{rad/s}$

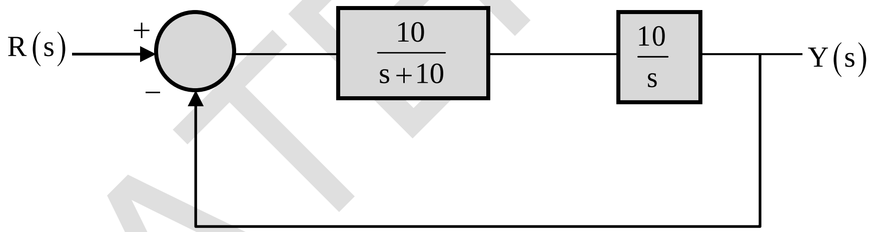
(B) $\zeta=0.1$ and $\omega_n=10\ \text{rad/s}$

(C) $\zeta=0.707$ and $\omega_n=10\ \text{rad/s}$

(D) $\zeta=0.707$ and $\omega_n=100\ \text{rad/s}$

Key: (A)

Sol: → The above block diagram can be redrawn as



$$G(s)=\frac{100}{s^2+10s}$$

→ Closed loop transfer function is: $\dfrac{100}{s^2+10s+100}$

Comparing with standard form: $k\dfrac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

By comparison

$\omega_n^2 = 100 \Rightarrow \omega_n = 10$

$2\xi\omega_n = 10 \Rightarrow \xi = \frac{10}{2\omega_n} = \frac{10}{20} = 0.5$

So, $\xi = 0.5$ and $\omega_n = 10$

40. e^A denotes the exponential of a square matrix A. Suppose λ is an eigenvalue and v is the corresponding eigen-vector of matrix A.

Consider the following two statements:

Statement 1: e^λ is an eigenvalue of e^A

Statement 2: v is an eigen-vector of e^A

Which one of the following options is correct?

(A) Statement 1 is true and statement 2 is false.

(B) Statement 1 is false and statement 2 is true.

(C) Both the statements are correct.

(D) Both the statements are false.

Key: (C)

Given λ is an eigen value of A.

Then eigen value of $\left(e^A = 1 + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \ldots\right)$ is

$\left(1 + \lambda + \frac{\lambda^2}{2!} + \ldots = e^\lambda\right)$

Hence statement 1 is true.

Since, we know that eigen vector of A and polynomial matrix A is same.

$\therefore$ Eigen vector of A and e^A is same

Hence statement (2) is true.

41. Let $f(x) = \int_0^x e^t (t-1)(t-2)\,dt$. Then f(x) decreases in the interval

(A) $x \in (1,2)$ (B) $x \in (2,3)$ (C) $x \in (0,1)$ (D) $x \in (0.5,1)$

Key: (A)

Sol: $f(x) = \int_0^x e^t (t-1)(t-2)\,dt$

$$\Rightarrow \quad f'(x) = \frac{d}{dx}(f(x))$$

$$= \frac{d}{dx}\left[\int_0^x e^t (t-1)(t-2)dt\right]$$

$$= e^x (x-1)(x-2) \qquad \left(\text{Since, } \frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x)\right)$$

We know that $e^x > 0$ and (Using fundamental theorem of integral calculus)

$(x-1)(x-2) < 0 \text{ for } 1 < x < 2$

$\Rightarrow \quad e^x (x-1)(x-2) < 0 \text{ for } 1 < x < 2$

$\therefore \quad f'(x) < 0, \forall x \in (1,2)$

$\Rightarrow \quad f(x)$ decrease in the interval $x \in (1,2)$. Option (A)

42. Consider a matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & 1 & 1 \end{bmatrix}$.

The matrix A satisfies the equation $6A^{-1} = A^2 + cA + dI$, where c and d are scalars and I is the identity matrix.

Then (c + d) is equal to

(A) 5 (B) 17 (C) –6 (D) 11

Key: (A)

Sol: The characteristic equation of A is $\begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 4-\lambda & -2 \\ 0 & 1 & 1-\lambda \end{vmatrix} = 0$

$\Rightarrow \quad (1-\lambda)\left[(4-\lambda)(1-\lambda)+2\right] = 0$

$\Rightarrow \quad \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

$\Rightarrow \quad A^3 - 6A^2 + 11A - 6I = 0,$ (Using Cayley-Hamilton theorem)

Multiplying both sides by A^{-1}, we get

$A^2 - 6A + 11I - 6A^{-1} = 0$

$\Rightarrow \quad 6A^{-1} = A^2 - 6A + 11I$

$\therefore \quad c = -6, d = 11 \Rightarrow c + d = 5$

Option (A)

43. The fuel cost functions in rupees/hour for two 600 MW thermal power plants are given by

Plant 1: $C_1 = 350 + 6P_1 + 0.004P_1^2$

Plant 2: $C_2\ 450 + aP_2 + 0.003P_2^2$

where P_1 and P_2 are power generated by plant 1 and plant 2, respectively, in MW and a is constant. The incremental cost of power (λ) is 8 rupees per MWh. The two thermal power plants together meet a total power demand of 550 MW. The optimal generation of plant 1 and plant 2 in MW, respectively, are

(A) 200, 350 (B) 250, 300 (C) 325, 225 (D) 350, 200

Key: **(B)**

Sol: Given,

Fuel cost function

$C_1 = 350 + 6P_1 + 0.004P_1^2$ where a is constant

$C_2 = 450 + aP_2 + 0.003\,P_2^2$

λ (incremental cost of power) = 8 Rs/MWh

Total load = 550 mW.

We need to find optimal generation of plant 1 and Plant 2 in MW

$I_{C_1} = 6 + 0.008\,P_1$ …(1) and

$I_{C_2} = 0.006\,P_2 + 0$

$\therefore\ \lambda = I_{C_1} = I_{C_2} = 8$ given (for optimal generation $I_{C_1} = I_{C_2} = \lambda$)

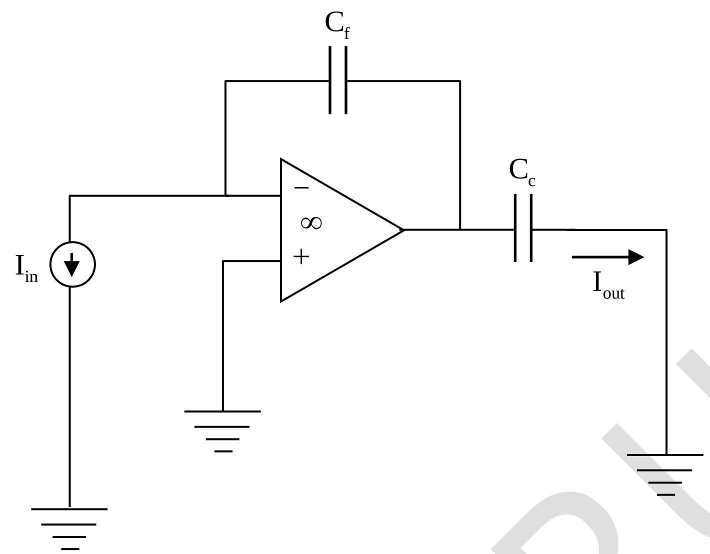
$\therefore\ 0.008P_1 + 6 = 8$

$\therefore\ P_1 = 250\ \text{MW}$

$\therefore\ P_2 = 550 - 250 = 300\ \text{MW}$

Hence option (B) is correct

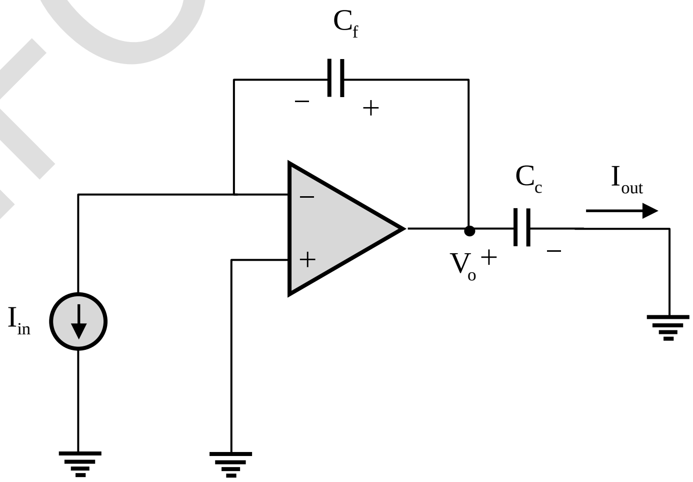
44. The current gain (I_{out}/I_{in}) in the circuit with an ideal current amplifier given below is



(A) $\frac{C_f}{C_c}$ (B) $\frac{-C_f}{C_c}$ (C) $\frac{C_c}{C_f}$ (D) $\frac{-C_c}{C_f}$

Key: **(C)**

Sol: op-amp is ideal, So $V_+ = V_-$



$$I_{out} = \frac{V_o}{\frac{1}{SC_C}} = V_o SC_C \qquad \ldots(1)$$

$$I_{in} = \frac{V_o}{\frac{1}{SC_f}} = V_o SC_f \qquad \ldots(2)$$

From equation (1) and (2)

$$\boxed{\frac{I_{out}}{I_{in}} = \frac{C_C}{C_f}}$$

45. If the magnetic field intensity (H) in a conducting region is given by the expression,

$H = x^2\hat{i} + x^2y^2\hat{j} + x^2y^2z^2\hat{k}\ A/m$. The magnitude of the current density, in A/m , at x = 1 m, y = 2 m, and z = 1 m, is

(A) 8 (B) 12 (C) 16 (D) 20

Key: (B)

Given, $\vec{H} = x^2\hat{i} + x^2y^2\hat{j} + x^2y^2z^2\hat{k}$ A/m, we need to calculate magnitude of the current density

$x = 1m, y = 2m \;\&\; z = 1$

As $\vec{J} = \nabla \times \vec{H}$

$$\vec{J} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & x^2y^2 & x^2y^2z^2 \end{vmatrix} = 2x^2yz^2\hat{a}_x - 2xy^2z^2\hat{a}_y + 2xy^2\hat{a}_z$$

$$\therefore \vec{J}\Big|_{1,2,1} = 2(1)(2)(1)\hat{a}_z - 2(1)(2)^2(1)^2\hat{a}_y + 2(1)(2)^2\hat{a}_z$$

$$\Rightarrow \vec{J} = 4\hat{a}_x - 8\hat{a}_y + 8\hat{a}_z$$

$$\therefore |\vec{J}| = \sqrt{(4)^2 + (8)^2 + (8)^2} = 12$$

Hence, the option (B) is correct.

46. Let a causal LTI system be governed by the following differential equation

$y(t) + \frac{1}{4}\frac{dy}{dt} = 2x(t)$, where $x(t)$ and $y(t)$ are the input and output respectively.

Its impulse response is

(A) $2e^{-\frac{1}{4}t}u(t)$ (B) $2e^{-4t}u(t)$ (C) $8e^{-\frac{1}{4}t}u(t)$ (D) $8e^{-4t}u(t)$

Key: **(D)**

Sol: $\rightarrow$ The given differential equation is

$$y(t) + \frac{1}{4}\frac{dy}{dt} = 2x(t)$$

By Laplace

$$\Rightarrow Y(s) + \frac{1}{4}Y(s) = 2X(s)$$

$$\Rightarrow \frac{Y(s)}{X(s)} = \frac{2}{1 + \frac{s}{4}}$$

$$\Rightarrow H(s) = \frac{8}{s+4}$$

$$\Rightarrow h(t) = 8e^{-4t}u(t)$$

So the impulse response $h(t) = L^{-1}(H(s))$ is $8e^{-4t}u(t)$

47. Let an input $x(t) = 2\sin(10\pi t) + 5\cos(15\pi t) + 7\sin(42\pi \tau) + 4\cos(45\pi t)$ is passed through an LTI system having an impulse response,

$$h(t) = 2\left(\frac{\sin(10\pi t)}{\pi t}\right)\cos(40\pi t)$$

The output of the system is

(A) $2\sin(10\pi t) + 5\cos(15\pi t)$ (B) $5\cos(15\pi t) + 7\sin(42\pi t)$

(C) $7\sin(42\pi t) + 4\cos(45\pi t)$ (D) $2\sin(10\pi t) + 4\cos(45\pi t)$

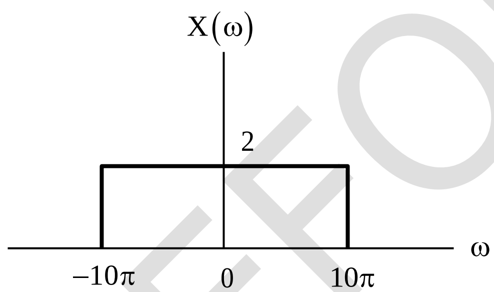
Key: **(C)**

Sol: → We know

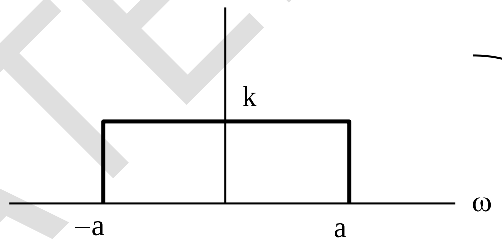
$$x(t)\cos\omega_o t \leftrightarrow \frac{1}{2}\left[X(\omega-\omega_o) + X(\omega+\omega_o)\right]$$

→ If we let $x(t) = 2\frac{\sin 10\pi t}{\pi t}$

Then $X(\omega)$

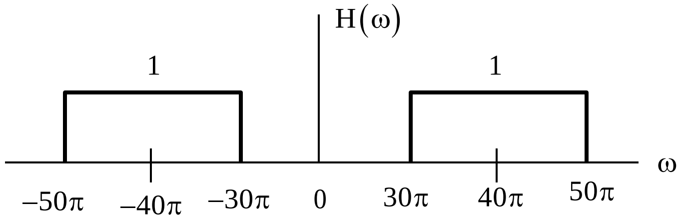


$\because k\frac{\sin at}{\pi t} \leftrightarrow$



→ $h(t) = x(t)\cos 40\pi t$

$$H(\omega) = \frac{1}{2}\left[X(\omega-40\pi) + X(\omega+40\pi)\right]$$



So impulse response represent a band pass filter with pass band ranging from $(30\pi \text{ to } 50\pi)$.

$\rightarrow$ Since given that

$$x(t) = \underset{\text{Stopband}}{2\sin 10\pi t} + \underset{\text{Stop band}}{5\cos 15\pi t} + \underset{\text{Pass band}}{7\sin 42\pi t} + \underset{\text{Pass band}}{4\cos 45\pi t}$$

Component lying in pass band range will be available at output

$$y(t) = 7\sin 42\pi t + 4\cos 45\pi t$$

48. Consider the system as shown below

where $y(t) = x(e^t)$. The system is

(A) linear and causal.
(B) linear and non-causal.
(C) non-linear and causal.
(D) non-linear and non-causal.

Key: (B)

Sol:

Input		Output
$x(t)$	$\rightarrow$	$x(e^t)$
$ax_1(t)$	$\rightarrow$	$ax_1(e^t)$
$bx_2(t)$	$\rightarrow$	$bx_2(e^t)$

$$ax_1(t) + bx_2(t) \rightarrow ax_1(e^t) + bx_2(e^t)$$

So the system is linear

$\rightarrow$ $y(t) = x(e^t)$

$y(0) = x(e^0) = x(1)$

Since present output depends on future value of input, the system is non causal.

49. The discrete time Fourier series representation of a signal $x[n]$ with period N is written as $x[n] = \sum_{k=0}^{N-1} a_k e^{j(2k\pi/N)}$. A discrete time periodic signal with period N = 3, has the non-zero Fourier series coefficient: $a_{-3} = 2$ and $a_4 = 1$. The signal is

(A) $2+2e^{-\left(j\frac{2\pi}{6}n\right)}\cos\left(\frac{2\pi}{6}n\right)$ (B) $1+2e^{\left(j\frac{2\pi}{6}n\right)}\cos\left(\frac{2\pi}{6}n\right)$

(C) $1+2e^{\left(j\frac{2\pi}{3}n\right)}\cos\left(\frac{2\pi}{6}n\right)$ (D) $2+2e^{\left(j\frac{2\pi}{6}n\right)}\cos\left(\frac{2\pi}{6}n\right)$

Key: (B)

Sol: → It is given that period of $x(n)$ is N = 3

$$x(n)=\sum_{k=0}^{2}a_k e^{j\frac{2kn\pi}{3}} \quad \ldots(1)$$

→ We know in DTFS $a_k=a_{k\pm\ell n}$

i.e., $a_k=a_{k\pm3}$

$a_{-3}=a_{-3+3}=a_0$ or a_{-6}

$a_4=a_{4\pm3}=a_1$ or a_7

i.e., $a_0=a_{-3}=2$

$a_1=a_4=1$

$a_2=0$ (given data)

→ Expanding equation (1)

$$x(n)=a_0+a_1e^{j\frac{2\pi}{3}n} \quad (a_2=0)$$

$$=2+e^{j\frac{2\pi}{3}n}$$

$$=1+e^{j\frac{2\pi}{6}n}\left[e^{j\frac{2\pi}{6}n}+e^{-j\frac{2\pi}{6}n}\right]$$

$$=1+2e^{j\frac{2\pi}{6}n}\cos\frac{2\pi}{6}n$$

50. Let, $f(x,y,z)=4x^2+7xy+3xz^2$. The direction in which the function f(x,y,z) increases most rapidly at point P = (1,0,2) is

(A) $20\hat{i}+7\hat{j}$ (B) $20\hat{i}+7\hat{j}+12\hat{k}$ (C) $20\hat{i}+12\hat{k}$ (D) $20\hat{i}$

Key: (B)

Sol: $\nabla f = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}$

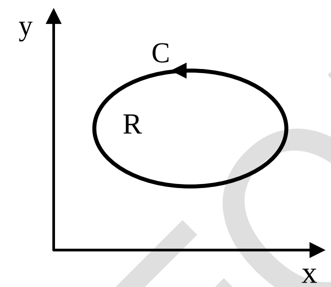
$$= \hat{i}\left(8x + 7y + 3z^2\right) + \hat{j}\left(7x\right) + \hat{k}\left(6xz\right)\Big|_{P(1,0,2)}$$

$$= \hat{i}\left(8 - 12\right) + \hat{j}\left(7\right) + \hat{k}\left(12\right) = 20\hat{i} + 7\hat{j} + 12\hat{k}$$

is the direction in which the function $f\left(x, y, z\right)$ increase most rapidly at point P = (0, 1, 2)

Option (B).

51. Let R be a region in the first quadrant of the xy plane enclosed by a closed curve C considered in counter-clockwise direction. Which of the following expressions does not represent the area of the region R?



(A) $\iint_R dxdy$ (B) $\oint_c xdy$ (C) $\oint_c ydx$ (D) $\frac{1}{2}\oint_c (xdy - ydx)$

Key: **(B, C)**

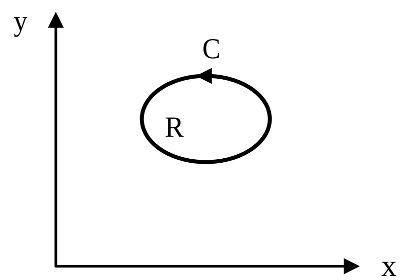
Sol: R is the region in xy plane enclosed by a closed curve C.

$\Rightarrow$ Area of the region R, using double integrals is $\iint_R dxdy$

Also from Green's theorem in vector calculus,

Area of the region R is $\frac{1}{2}\oint_c \left(xdy - ydx\right)$

$\therefore$ Option (B, C)



52. Let $\vec{E}(x, y, z) = 2x^2\hat{i} + 5y\hat{j} + 3z\hat{k}$. The value of $\iiint_V \left(\vec{\nabla}.\vec{E}\right)dV$, where V is the volume enclosed by the unit cube defined by $0 \le x \le 1, 0 \le y \le 1$, and $0 \le z \le 1$, is

(A) 3 (B) 8 (C) 10 (D) 5

Key: **(C)**

Sol: $\vec{\nabla}.\vec{E} = \text{div}\left(\vec{E}\right) = \frac{\partial}{\partial x}\left(2x^2\right) + \frac{\partial}{\partial y}\left(5y\right) + \frac{\partial}{\partial z}\left(3z\right)$

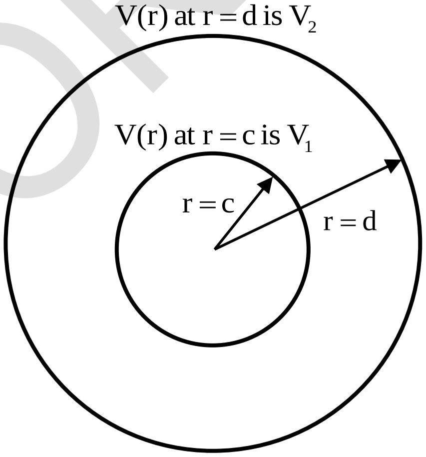
$= 4x + 5 + 3 = 4x + 8$

$\therefore \quad \iiint_v \left(\vec{\nabla}.\vec{E}\right) dV = \int_{x=0}^{1}\int_{y=0}^{1}\int_{z=0}^{1} \left(4x+8\right) dz dy dx$

$= \left(\int_{x=0}^{1} \left(4x+8\right) dx\right) \times \left(\int_{y=0}^{1} dy\right) \times \left(\int_{z=0}^{1} dx\right)$

$= \left(2x^2 + 8x\right)_0^1 . \left(y\right)_0^1 . \left(z\right)_0^1 = 10.$

53. As shown in the figure below, two concentric conducting spherical shells, centered at r = 0 and having radii r = c and r = d are maintained at potentials such that the potential V(r) at r = c is V_1 and V(r) at r = d is V_2. Assume that V(r) depends only on r, where r is the radial distance. The expression for V(r) in the region between r = c and r = d is



(A) $V\left(r\right) = \frac{cd\left(V_2 - V_1\right)}{(d-c)r} - \frac{V_1 c + V_2 d - 2V_1 d}{d-c}$

(B) $V(r) = \frac{cd(V_1 - V_2)}{(d-c)r} + \frac{V_2 d - V_1 c}{d-c}$

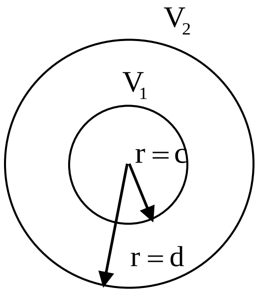
(C) $V(r) = \frac{cd(V_1 - V_2)}{(d-c)r} - \frac{V_1 c - V_2 c}{d-c}$

(D) $V(r) = \frac{cd(V_2 - V_1)}{(d-c)r} - \frac{V_2 c - V_1 c}{d-c}$

Key: (B)

Since Laplace's equation $\nabla^2 V = 0$

For spherical co-ordinate system,



$\nabla^2 V = \frac{1}{r^2 \sin\theta}\left\{\frac{\partial}{\partial r}\left[\frac{r^2 \sin\theta}{1}\frac{\partial V}{\partial r}\right]\right\} = 0$

$\therefore r^2 \frac{dV}{dr} = A = \text{constant}$

$\frac{dV}{dr} = \frac{A}{r^2}$

$$\Rightarrow V = \frac{-A}{r} + B, B = \text{constant}$$

At $r = C,\ V = V_1$, at $r = d,\ V = V_2$

$$\therefore \quad V_1 = \frac{A}{c} + B. \qquad ...(1)$$

$$V_2 = \frac{-A}{d} + B \qquad ...(2)$$

By Equation (1) and (2) we get

$$V_1 - V_2 = \frac{-A}{c} + \frac{A}{d} \Rightarrow V_1 - V_2 = A\left[\frac{c-d}{cd}\right]$$

$$\therefore A = \left[\frac{V_1 - V_2}{c-d}\right]cd$$

From (1), $B = V_1 + \frac{A}{C}$

$$B = V_1 + \frac{(V_1 - V_2)}{(c-d)}d$$

$$\therefore V = \frac{-\left(\frac{V_1 - V_2}{c-d}\right)cd}{r} + V_1 + d\left(\frac{V_1 - V_2}{c-d}\right)$$

$$\therefore \boxed{V(r) = \frac{cd(V_1 - V_2)}{(d-c)r} + \frac{V_2 d - V_1 c}{d-c}}$$

54. Let the probability density function of a random variable x be given as

$$f(x) = ae^{-2|x|}$$

The value of 'a' is ___________ .

Key: **(1)**

Sol: Since, $f(x)$ is probability density function

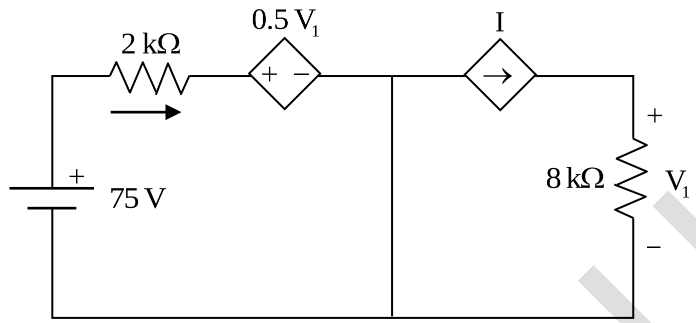
$$\therefore \quad \int_x f(x)dx = 1 \Rightarrow \int_{-\infty}^{\infty} ae^{-2|x|}dx = 1 \text{ (here x takes values from } -\infty \text{ to } \infty)$$

$$\Rightarrow \quad a \times 2\int_0^{\infty} e^{-2|x|}dx = 1 \qquad \left(\text{since } e^{-2|x|} \text{ is even function}\right)$$

$$\Rightarrow \quad 2a\int_0^{\infty} e^{-2x}\,dx = 1 \qquad \left(\text{Since, } |x| = x \text{ if } x > 0\right)$$

$$\Rightarrow \quad 2a\left(\frac{e^{-2x}}{-2}\right)_0^{\infty} = 1 \Rightarrow a\left(e^0 - e^{-\infty}\right) = 1 \Rightarrow a = 1$$

55. In the circuit shown below, the magnitude of the voltage V in volts, across the $8k\Omega$ resistor is ______________. (round off to nearest integer)



Key: (100)

Sol: → From the circuit we can say

$$V_1 = 8I \text{ or } I = \frac{V_1}{8}$$

→ By KVL in left side mesh we have

$$75 - 2I - 0.5V_1 = 0$$

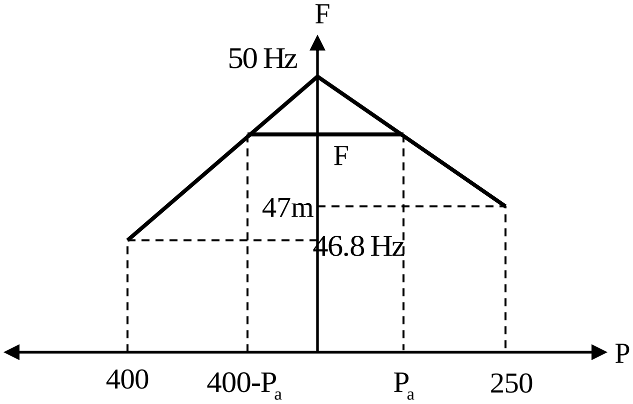
$$\Rightarrow 75 - 2\left(\frac{V_1}{8}\right) - \left(\frac{V_1}{2}\right) = 0 \Rightarrow \frac{V_1}{4} + \frac{V_1}{2} = 75 \Rightarrow V_1 + 2V_1 = 300 \Rightarrow V_1 = \frac{300}{3} = 100V$$

56. Two generating units rated for 250 MW and 400 MW have governor speed regulations of 6% and 6.4%, respectively, from no load to full load. Both the generating units are operating in parallel to share a load of 500 MW. Assuming free governor action, the load shared in MW, by the 250 MW generating unit is _________. (round off to nearest integer)

56. (200)

From the given information in question

From the above plot



$$\frac{3}{250} = \frac{50 - f}{P_a} \qquad \ldots(i)$$

$$\frac{3.2}{400} = \frac{50 - f}{500 - Pa} \qquad \ldots(ii)$$

After solving equation (i) and (ii)

$$\boxed{P_a = 200 \text{ MW}}$$

∴ This is the load shared by 250 MW generating unit.

57. A 20 MVA, 11.2 kV, 4-pole, 50 Hz alternator has an inertia constant of 15 MJ/MVA. If the input and output powers of the alternator are 15 MW and 10 MW, respectively, the angular acceleration in mechanical degree/s^2 __________. (round off to nearest integer)

Key: (75)

Sol: Given, 20 MVA, 11.25V, 4 pole, 50 Hz, H = 15 MJ/MVA, $P_{in} = 15\,MW$, $P_{out} = 10\,MW$; We need to calculate angular acceleration in mechanical degree/s^2.

$$M\frac{d^2\delta}{dt^2} = P_a = 15 - 10 = 5$$

Where $M = \frac{GH}{\pi f}$

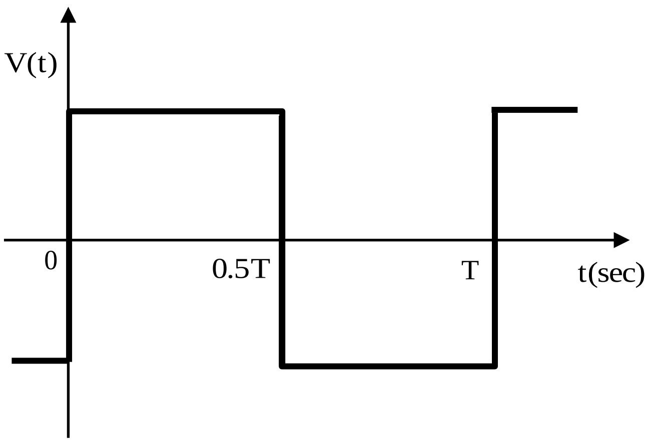
$$\therefore \quad \underset{\alpha}{\underset{\downarrow}{\frac{d^2\delta}{dt^2}}} = \frac{5}{M} = \frac{15\times 20}{180\times 50} = \frac{1}{30}$$

$$\therefore \quad \text{Acceleration } \alpha = \frac{5}{\frac{1}{30}} = 150 \text{ electrical degree/sec}^2$$

But we have to calculate α in Mechanical degree/$\sec^2$

$$\therefore \quad \alpha = 150\times\frac{2}{P} = 150\times\frac{2}{4} = 75 \text{ Mechanical degree/sec}^2$$

58. Consider an ideal full-bridge single-phase DC-AC inverter with a DC bus voltage magnitude of 1000 V. The inverter output voltage (t) shown below, is obtained when diagonal switches of the inverter are switched with 50 % duty cycle. The inverter feeds a load with a sinusoidal current given by, $i(t) = 10\sin\left(\omega t - \frac{\pi}{3}\right)A$, where $\omega = \frac{2\pi}{T}$.



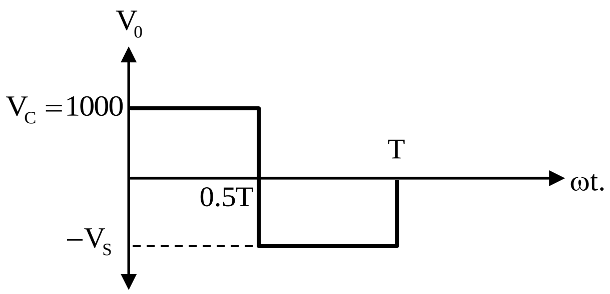
The active power, in watts, delivered to the load is __________.(round off to nearest integer)

Key: (3182)

Sol: Given, $V_{dc} = 1000\,V$ (Input to the single phase inverter)

Duty cycle D = 50%, $i(t)_{ouput} = 10\sin\left(\omega t - \frac{\pi}{3}\right)A$;

Where $\omega = \frac{2\pi}{T}$. We need to calculate active power in watts.



Since current is having fundamental component only. Therefore to calculate active power, we have to consider V_o fundamental component only

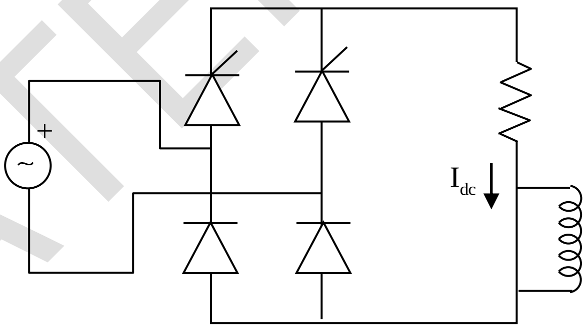
$$\therefore \quad P_0 = P_{01} = V_{01}.I_{or} \cos\phi$$

$$= \frac{2\sqrt{2}}{\pi} \times 1000.\frac{10}{\sqrt{2}} \times 0.5$$

$$\simeq 3182W$$

Hence, the power in watts, delivered to the load is 3182W.

59. For the ideal AC-DC rectifier circuit shown in the figure below, the load current magnitude is $I_{dc} = 15A$ and is ripple free. The thyristors are fired with a delay angle of 45^o The amplitude of the fundamental component of the source current, in amperes, is __________. (round off to two decimal places)



Key: (17.65A)

Sol: Given, AC-DC rectifier, and $I_{dc} = 15A$ and L is sufficient to make current ripple free $\alpha = 45°$.

$\therefore$ The fundamental component of source current

$$= \frac{4I_{dc}}{\eta\pi}\cos\left(\frac{\alpha n}{2}\right), \text{ where } n = 1 = \frac{4I_{dc}}{\pi}\cos\frac{\alpha}{2}$$

$$I_s = \frac{4\times15}{\pi}\cos\left(\frac{45}{2}\right) = 17.65A$$

$\therefore$ The amplitude of I_{s_1} is 17.65A.

60. A 3-phase grid-connected voltage source converter with DC link voltage of 1000 V is switched using sinusoidal Pulse Width Modulation (PWM) technique. If the grid phase current is 10 A and the 3-phase complex power supplied by the converter is given by $(-4000 - j3000)$ VA, then the modulation index used in sinusoidal PWM is ___________. (round off to two decimal places)

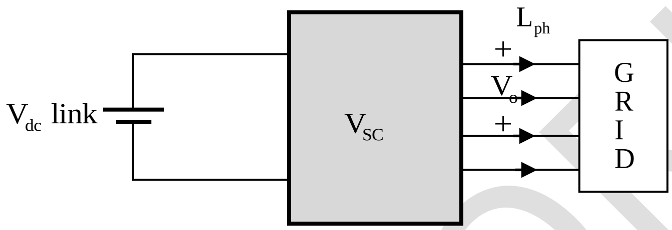
Key: **(0.47)**

Sol: V_{dc} link = 1000V.

$I_{ph} = 10A$

$S_{complex\ power} = (-4000 - j3000)\,VA$

We need to calculate the modulation index used in sinusoidal PMW.



$$P_0 = \sqrt{3} V_{L_1} I_{or} (p-f)$$

$$4000 = \sqrt{3} \times V_{L_1} \times 10 \left\{ \frac{4000}{\sqrt{4000^2 + (3000)^2}} \right.$$

$$\therefore V_{L_1} rms = 288.675V$$

$$\therefore 3\phi\ V_{SI} - SPWH\ \mu_A \le 1$$

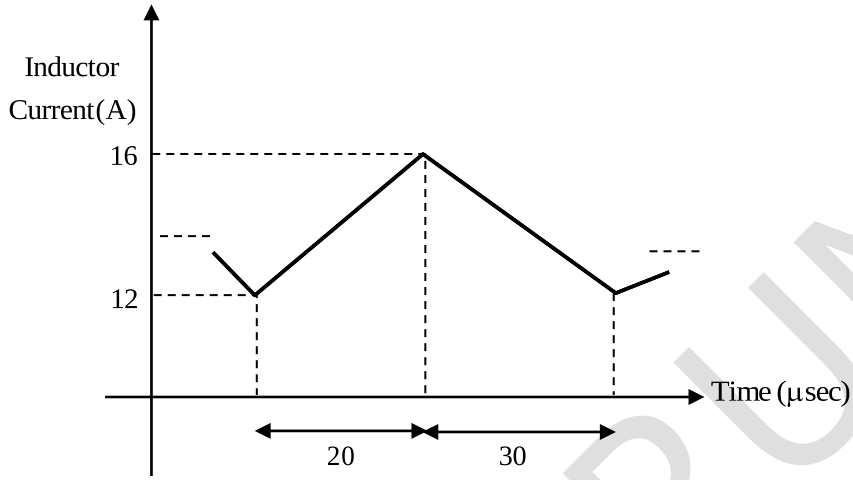
$$\therefore V_{L1} = \sqrt{3} \mu_A . \frac{V_S}{2}$$

$$V_{L_1(rms)} = \mu_A \left\{ \frac{\sqrt{3}}{2\sqrt{2}} \times 1000 \right\}$$

$$\therefore \boxed{\mu_A = 0.47}$$

Hence, the modulation index is 0.47

61. The steady state current flowing through the inductor of a DC-DC buck boost converter is given in the figure below. If the peak-to peak ripple in the output voltage of the converter is 1V, then the value of the output capacitor, in μF, is ___________. (round off to decimal integer)



Key: **(168)**

Sol: Given, V_{pp} ripple = 1V

We need to calculate the value of the output capacitor in μF,

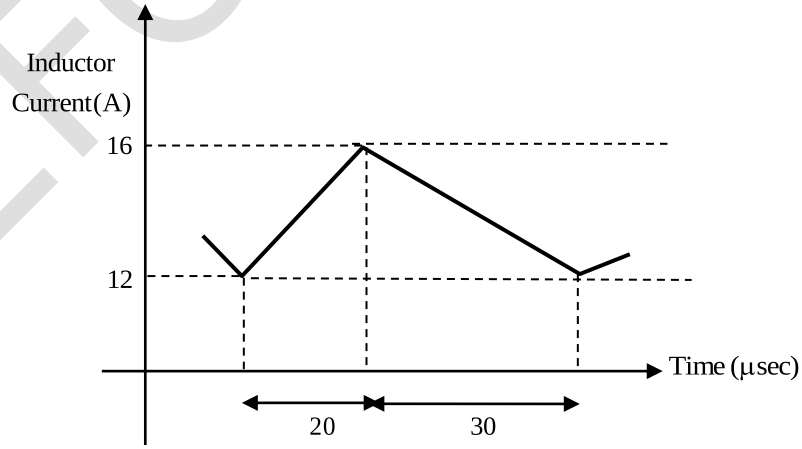
$$I_{L\,avg} = \frac{I_{max} + I_{min}}{2} = \frac{16+12}{2} = 14A$$

$$I_o = I_L\{1-\alpha\} \text{ for buck} - \text{boost converter}$$

$$I_o = 14\left\{1 \times \frac{30}{50}\right\} = 8.4A$$

$$\Delta V_o = \Delta V_C = \frac{\alpha I_o}{FC} \Rightarrow 1 = \frac{\frac{2}{5} \times 8.4}{\frac{1}{50 \times 10^{-6}} \times C}$$

$$\therefore C = 168\,\mu F$$



62. A 280 V, separately excited DC motor with armature resistance of 1Ω and constant field excitation drives a load. The load torque is proportional to the speed. The motor draws a current of 30 A when running at a speed of 1000 rpm. Neglect frictional losses in the motor. The speed, in rpm, at which the motor will run, if an additional resistance of value 10Ω is connected in series with the armature, is __________. (round off to nearest integer)

Key: (483)

Sol: Given, $V_{dc} = 280\,V$, separately excited motor, $R_a = 1\Omega$, field is constant (ϕ -constant). $T \propto N \,\&\, T_a \propto I_a = 30 N = 1000 rpm, R_{ext} = 10\Omega$

Since $\phi =$ constant

$$\therefore \frac{T_2}{T_1} = \frac{I_{a_2}}{I_{a_2}} = \frac{N_2}{N_1}$$

$$\therefore \frac{I_{a_2}}{I_{a_1}} = \frac{N_2}{N_1} \Rightarrow \therefore \boxed{I_{a_2} = \frac{30}{1000} N_2}$$

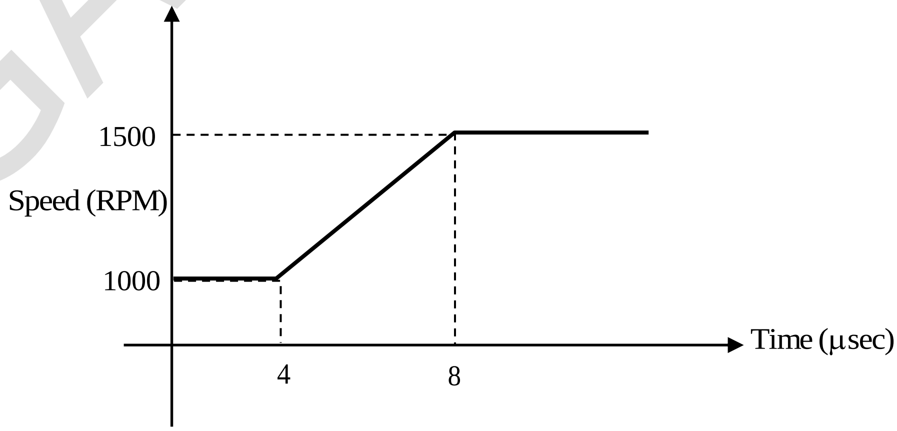
$$\therefore \boxed{I_{a_2} = 0.03\, N_2}$$

$$\therefore N \alpha \frac{\in_t}{\phi} \Rightarrow \frac{N_2}{N_1} = \frac{V - I_{a_2}\left(R_a + R_{ext}\right)}{V - I_a R_a}$$

$$\Rightarrow \frac{N_2}{1000} = \frac{280 - \left(0.03 N_2\right)\left(1 + 10\right)}{280 - 30} \Rightarrow N_2 = 482.76 \text{ rpm}$$

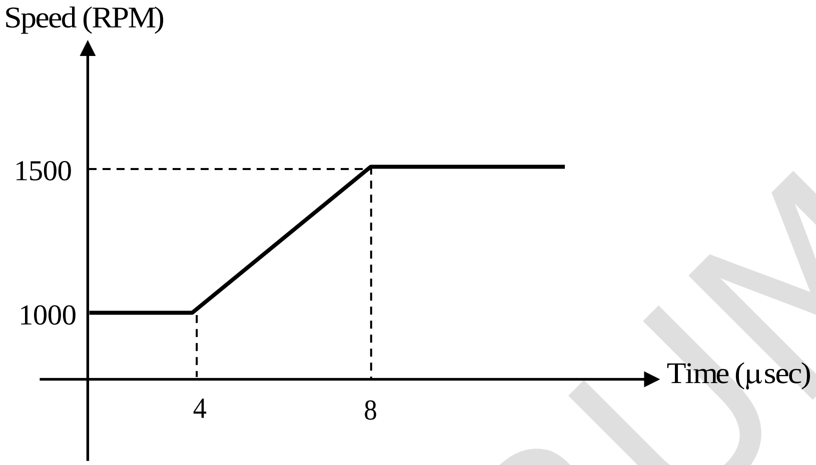
$\therefore$ The speed in rpm where $r_{ext} = 10\,\Omega$ is connected is 483 rpm (rounded off to nearest integer).

63. A 4-pole induction motor with inertia of $0.1\,kg\text{-}m^2$ drives a constant load torque of 2 Nm. The speed of the motor is increased linearly from 1000 rpm to 1500 rpm in 4 seconds as shown in the figure below. Neglect losses in the motor. The energy, in joules, consumed by the motor during the speed change is ____________. (round off to nearest integer)



Key: **(1733)**

Sol: Given, $P = 4$, $J = 0.1$ kg-m^2, $T_L = 2$ Nm(constant)



& we need to calculate the energy, in Joules consumed by the motor during the speed change.

$N(t) = 125t + 500$ valid for $4 < t < 8$ sec

$$\therefore \quad j\frac{d\omega}{dt} = T_e - T_L$$

$$T_e = i\frac{d\omega}{dt} + T_L$$

After multiply ω both since.

$$\omega T_e = j\omega\frac{d\omega}{dt} + \omega T_L \qquad \{\because P_e = \frac{dE}{dt} = \text{change in energy with respect to time})$$

$$\Rightarrow P_e = j\omega\frac{d\omega}{dt} + \omega T_L$$

$$\Rightarrow \frac{dE}{dt} = j\omega\frac{d\omega}{dt} = \omega T_L$$

$$\Rightarrow dE = j\omega + \omega + \omega T_L dt$$

$$E = j\int \omega d\omega + T_L \int \omega dt$$

$$E = j\left[\frac{\omega^2}{2}\right]_{\omega_2}^{\omega_1} + P \qquad \omega_0 = \frac{2\pi N}{60} = \frac{2000\pi}{60}, \omega_1 = \frac{300\pi}{60}$$

$$= 0.1\left[\left(\frac{2\pi \times 1000}{60^2 \times 2}\right)^2 - \left[\frac{(3000\pi)^2}{60^2 \times 2}\right]\right]$$

$$E = 685.39 \text{ joules} + P \rightarrow (\text{just a variable})$$

$$P = T_L \int \omega dt = T_L \frac{2\pi}{60} \int N dt = T_L \frac{2\pi}{60} \int_4^8 (25t + 500) dt$$

$$P = 2 \times \frac{2\pi}{60} \left[\frac{125t^2}{2} + 500t \right]_4^8 = 1047.197 \text{ joules}$$

$$E_T = 1732.586 \text{ joules}$$

64. A star-connected 3-phase, 400 V, 50 kVA, 50 Hz synchronous motor has a synchronous reactance of 1 ohm per phase with negligible armature resistance. The shaft load on the motor is 10 kW while the power factor is 0.8 leading. The loss in the motor is 2 kW. The magnitude of the per phase excitation emf of the motor, in volts, is __________. (round off to nearest integer).

Key: **(245)**

Sol: $V_{L-L} = 400\text{V}, 50\text{ kVA}, 50\text{ Hz}$ Synchronous motor given

$X_L = 1\Omega/\text{ph}.\ r_a \simeq 0\Omega,\ P_{sh} = 10\text{ kW},\ \text{pf} = 0.8$ leading.

$P_{loss} = 2\text{kW., Emf} = ?$ (need to calculate)

For Motor

$$E_b = \sqrt{(V_{ph}\cos\phi - I_a R_a)^2 + (V\sin\phi \pm I_a X_a)^2}$$

For leading (+), For lagging (−)

Where, $I_a = \frac{\text{Input Power}}{\sqrt{2}\ V_L \times \text{pf}} = \frac{12\text{ kW}}{\sqrt{3} \times 400 \times 0.8} = 21.65\text{A}$

$$\therefore E_b = \sqrt{\left(\frac{400}{\sqrt{3}} \times 0.8 - 0\right)^2 + \left[\frac{400}{\sqrt{3}} \times 0.6 + 21.65 \times 1\right]^2}$$

....

Since pf is leading (+)

$$\therefore \boxed{E_b = 244.5\text{ V/ph}}$$

Need to take of question last line, E_b is asked in per phase only. Hence correct answer is 244.5V.

65. A 3-phase, 415 V, 4-pole, 50 Hz induction motor draws 5 times the rated current at rated voltage at starting. It is required to bring down the starting current from the supply to 2 times of the rated current using a 3-phase autotransformer. If the magnetizing impedance of the induction motor and no load current of the autotransformer is neglected, then the transformation ratio of the autotransformer is given by ____________. (round off to two decimal places).

Key: **(0.63)**

Sol: Given, $I_{SC} = 5I_{\text{rated}}$

$\therefore \boxed{I_{sc} = 5I_{FL}}$, we need to limit

Starting current $2\,I_{FL}$ using auto-transformer

$\therefore \boxed{I_L = x^2 I_{SC}} \rightarrow$ For auto-transformer I_L and I_{SC} relationship.

$\therefore 2I_{FL} = x^2 I_{FL} = x^2 5I_{FL}$

$\therefore x^2 = \frac{2}{5} \Rightarrow x = \sqrt{\frac{2}{5}} = 0.63$

Hence the transformer ratio of the auto-transformer is given by 0.63.