GENERAL APTITUDE

Q. No. 1-5 Carry One Mark Each

1. The movie was funny and I _____________.

(A) could help laughing
(B) could helped lauged
(C) couldn't help lauged
(D) couldn't help lauging

2. $x : y : z = \frac{1}{2} : \frac{1}{3} : \frac{1}{4}$.

What is the value of $\frac{x+z-y}{y}$?

(A) 1.25
(B) 3.25
(C) 0.75
(D) 2.25

3. Both the numerator and the denominator of $\frac{3}{4}$ are increased by a positive integer, x, and those of $\frac{15}{17}$ are decreased by the same integer. This operation results in the same value for both the fraction.

What is the value of x?

(A) 1
(B) 3
(C) 4
(D) 2

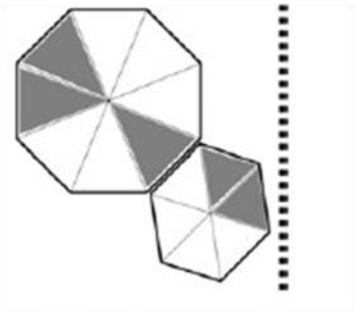
4. A survey of 450 students about their subjects of interest resulted in the following outcome.

- 150 students are interested in Mathematics.
- 200 students are interested in Physics.
- 175 students are interested in Chemistry.
- 50 students are interested in Mathematics and Physics.
- 60 students are interested in Mathematics and Chemistry.
- 30 students are interested in Mathematics, Physics and Chemistry.
- Remaining students are interested in Humanities.

Based on the above information, the number of students interested in Humanities is

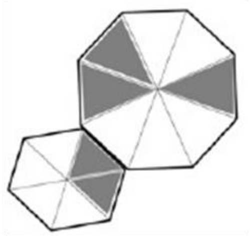
(A) 10
(B) 45
(C) 40
(D) 30

5.

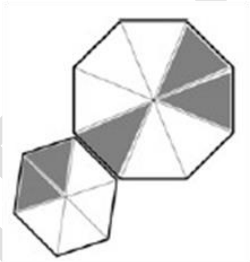


For the picture shown above, which one of the following is the correct picture representing reflection with respect to the mirror shown as the dotted line?

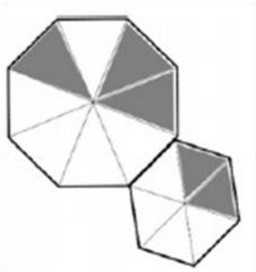
(A)



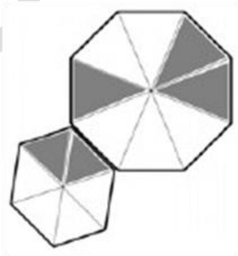
(B)



(C)



(D)



Q. No. 6-10 Carry Two Marks Each

6. In the last few years, several new shopping malls were opened in the city. The total number of visitors in the malls is impressive. However, the total revenue generated through sales in the shops in these malls is generally low.

Which one of the following is the CORRECT logical inference based on the information in the above passage?

(A) More people are visiting the malls and spending more

(B) Fewer people are visiting the malls but spending more

(C) More people are visiting the malls but not spending enough

(D) Fewer people are visiting the malls and not spending enough

7. In a partnership business the monthly investment by three friends for the first six months is in the ratio 3:4:5. After six months, they had to increase their monthly investment by 10%, 15% and 20% respectively, of their initial monthly investment. The new investment ration was kept constant for the next six months.

What is the ratio of their shares in the total profit (in the same order) at the end of the year such that the share is proportional to their individual total investment over the year?

(A) 33: 46: 60 (B) 22: 33: 50 (C) 22: 23: 24 (D) 63: 86: 110

8. Consider the following equations of straight lines:

Line L1: $2x - 3y = 5$

Line L2: $3x + 2y = 8$

Line L3: $4x - 6y = 5$

Line L4: $6x - 9y = 6$

Which one among the following is the correct statement?

(A) L1 is parallel to L2 and L1 is perpendicular to L3

(B) L2 is parallel to L4 and L2 is perpendicular to L1

(C) L3 is perpendicular to L4 and L3 is parallel to L2

(D) L4 is perpendicular to L2 and L4 is parallel to L3

9. Given below are two statements and four conclusions drawn based on the statements.

Statement 1: Some soaps are clean.

Statement 2: All clean objects are w**et.**

Conclusion I: Some clean objects are soaps.

Conclusion II: No clean object is a soap.

Conclusion III: Some wet objects are soaps.

Conclusion IV: All wet objects are soaps.

Which one of the following options can be logically inferred?

(A) Either conclusion III or conclusion IV is correct

(B) Either conclusion I or conclusion II is correct

(C) Only conclusion I is correct

(D) Only conclusion I and conclusion III are correct

10. An ant walks in a straight line on a plane leaving behind a trace of its movement. The initial position of the ant is at point P facing east.

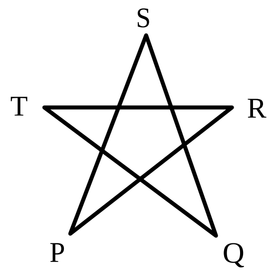
The ant first turns 72° anticlockwise at P, and then does the following two steps in sequence exactly FIVE times before halting.

1. Moves forward for 10 cm.

2. Turns 144° clockwise.

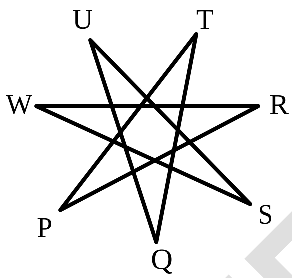
The pattern made by the trace behind by the ant is

(A)



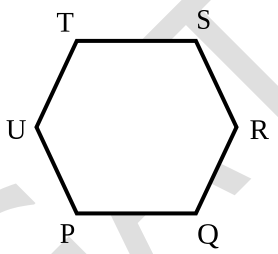
$SQ = QT = TR = RP = PS = 10\,cm$

(B)



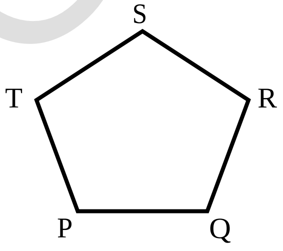
$SW = WR = RP = PT = TQ = QU = US = 10\,cm$

(C)



$PQ = QR = RS = ST = TU = UP = 10\,cm$

(D)



$PQ = QR = RS = ST = TP = 10cm$

CIVIL ENGINEERING

Q. No. 11-35 Carry One Mark Each

11. The function f(x, y) satisfies the Laplace equation $\nabla^2 f(x,y) = 0$ on a circular domain of radius r = **1** with its center at point P with coordinates x = 0. y=0. The value of this function on the circular boundary of this domain is equal to 3.

The numerical value of f(0, 0) is :

(A) 0 (B) 2 (C) 3 (D) 1

Key: (C)

Sol: Since f(x, y) satisfies the Laplace's equations on a circular domain of radius r = 1 with center at (0, 0) and value of f(x, y) is 3 for all points (x, y) on the circular boundary of this domain

$\therefore \quad f(x,y) = 3$ is a constant function

$\therefore \quad f(0,0) = 3$

12. $\int \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ... \right) dx$ is equal to

(A) $\frac{1}{1+x}$ + constant (B) $\frac{1}{1+x^2}$ + constant (C) $-\frac{1}{1-x}$ + constant (D) $-\frac{1}{1-x^2}$ + constant

Key: (*)

Sol: $\int \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ... \right) dx = \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{12} - \frac{x^5}{20} + ... + C$

Now (A) $\frac{1}{1+x} = (1+x)^{-1} = 1 - x + x^2 - x^3 + ...$

(B) $\frac{1}{1+x^2} = (1+x^2)^{-1} = 1 - x^2 + (x^2)^2 - (x^2)^3 + ... = 1 - x^2 + x^4 - x^6 + ...$

(C) $\frac{-1}{1-x} = -(1-x)^{-1} = -(1 + x + x^2 + x^3 + ...)$

(D) $-\frac{1}{1-x^2} = -(1-x^2)^{-1} = -(1 + x^2 + x^4 + x^6 + ...)$

$\therefore$ None of the options is the answer.

13. For a linear elastic and isotropic material, the correct relationship among Young's modulus of elasticity (E), Poisson's ratio (ν), and shear modulus (G) is

(A) $G = \frac{E}{2(1+\nu)}$ (B) $G = \frac{E}{(1+2\nu)}$ (C) $E = \frac{G}{2(1+\nu)}$ (D) $E = \frac{G}{(1+2\nu)}$

Key: **(A)**

Sol: We know that

$$E = 2G(1+\nu)$$

$$G = \frac{E}{2(1+\nu)}$$

14. Read the following statements relating to flexure of reinforced concrete beams:

I. In over-reinforced sections, the failure strain in concrete reaches earlier than the yield strain in steel.

II. In under-reinforced sections, steel reaches yielding at a load lower than the load at which the concrete reaches failure strain.

III. Over-reinforced beams are recommended in practice as compared to the under-reinforced beams.

IV. In balanced sections, the concrete reaches failure strain earlier than the yield strain in tensile steel.

Each of the above statements is either True or False.

Which one of the following combinations is correct?

(A) I (True), II (True), III (False), IV (False)
(B) I (True), II (True), III (False), IV (True)
(C) I (False), II (False), III (True), IV (False)
(D) I (False), II (True), III (True), IV (False)

Key: **(A)**

15. Match all the possible combinations between Column X (Cement compounds) and Column Y (Cement properties):

Column X	Column Y
(i) C_3S	**(P)** Early age strength
(ii) C_2S	**(Q)** Later age strength
(iii) C_3A	**(R)** Flash setting
	(S) Highest heat of hydration
	(T) Lowest heat of hydration

Which one of the following combinations is correct?

(A) (i) - (P), (ii) - (Q) and (T), (iii) - (R) and (S)

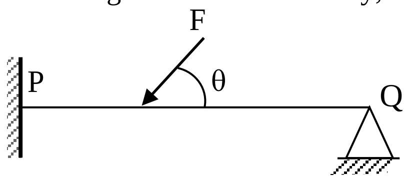
(B) (i) - (Q) and (T), (ii) - (P) and (S), (iii) - (R)

(C) (i) - (P), (ii) - (Q) and (R), (iii) - (T)

(D) (i) - (T), (ii) - (S), (iii) - (P) and (Q)

Key: **(A)**

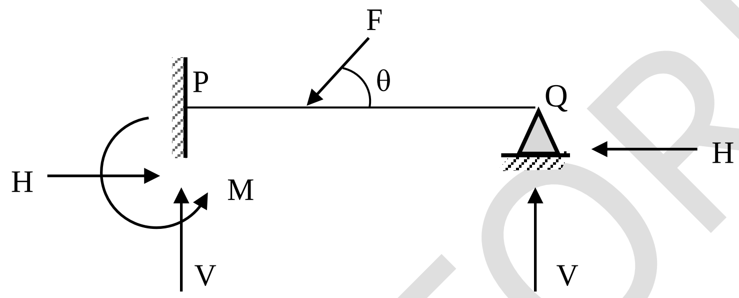
16. Consider a beam PQ fixed at P, hinged at Q, and subjected to a load F as shown in figure (not drawn to scale). The static and kinematic degrees of indeterminacy, respectively, are



(A) 2 and 1 (B) 2 and 0 (C) 1 and 2 (D) 2 and 2

Key: **(A)**

Sol:



Static indeterminacy = Number of unknown reactions – equilibrium reactions

$D_S = 5 - 3 = 2$

Kinematic indeterminacy $(D_k) = 1$ (rotation)

17. Read the following statements:

(P) While designing a shallow footing in sandy soil, monsoon season is considered for critical design in terms of bearing capacity.

(Q) For slope stability of an earthen dam, sudden drawdown is never a critical condition.

(R) In a sandy sea beach, quicksand condition can arise only if the critical hydraulic gradient exceeds the existing hydraulic gradient.

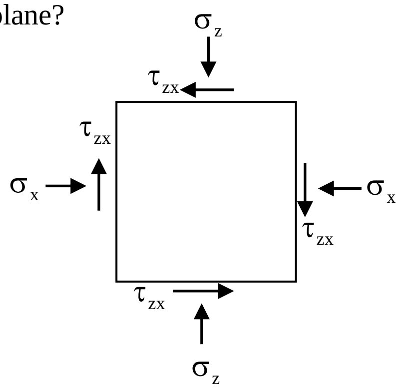
(S) The active earth thrust on a rigid retaining wall supporting homogeneous cohesionless backfill will reduce with the lowering of water table in the backfill.

Which one of the following combinations is correct?

(A) (P)-True, (Q)-False, (R)-False, (S)-False
(B) (P)-False, (Q)-True, (R)-True, (S)-True
(C) (P)-True, (Q)-False, (R)-True, (S)-True
(D) (P)-False, (Q)-True, (R)-False, (S)-False

Key: **(C)**

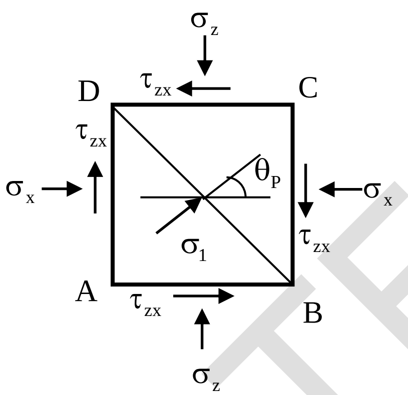
18. Stresses acting on an infinitesimal soil element are shown in the figure (with $\sigma_2 > \sigma_x$). The major and minor principal stresses are σ_1 and σ_3 respectively. Considering the compressive stresses as positive, which one of the following expressions correctly represents the angle between the major principal stress plane and the horizontal plane?



(A) $\tan^{-1}\left(\dfrac{\tau_{zx}}{\sigma_1 - \sigma_x}\right)$ (B) $\tan^{-1}\left(\dfrac{\tau_{zx}}{\sigma_3 - \sigma_x}\right)$ (C) $\tan^{-1}\left(\dfrac{\tau_{zx}}{\sigma_1 + \sigma_x}\right)$ (D) $\tan^{-1}\left(\dfrac{\tau_{zx}}{\sigma_1 + \sigma_3}\right)$

Key: (A)

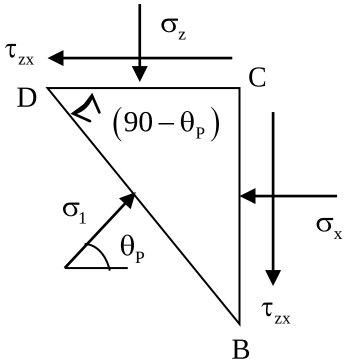
Sol:



Angle principle plane will be obtained as follows,

Let σ_1 = minimum principal stress making an angle 'θ_P' Horizontal plane i.e., with 'CD'

Taking 'BCD' portion of element, draw the free body diagram, it will be as follows. Assume unit thickness of element



Equating forces along x-direction to zero

$$\sigma_1(BD)\cos\theta_P - \sigma_x(BC) - \tau_{zx}(CD) = 0$$

$$\sigma_1\left(\frac{BC}{\cos\theta_P}\right)\cos\theta_P - \sigma_x BC - \tau_{zx}\left(\frac{BC}{\tan(90-\theta_P)}\right) = 0$$

$$\sigma_1 - \sigma_x = \tau_{zx}\left(\frac{1}{\tan(90-\theta_P)}\right)$$

$$\tan(90-\theta_P) = \frac{\tau_{xz}}{\sigma_1 - \sigma_x}$$

∴ Angle with horizontal plane i.e., with 'CD' is

$$(90-\theta_P) = \tan^{-1}\left(\frac{\tau_{zx}}{\sigma_1 - \sigma_x}\right)$$

So answer is option 'A'.

19. Match Column X with Column Y:

Column X	Column Y
(P) Horton equation	**(I)** Design of alluvial channel
(Q) Penman method	**(II)** Maximum flood discharge
(R) Chezy's formula	**(III)** Evapotranspiration
(S) Lacey's theory	**(IV)** Infiltration
(T) Dicken's formula	**(V)** Flow velocity

Which one of the following combinations is correct?

(A) (P)-(IV), (Q)-(III), (R)-(V), (S)-(I), (T)-(II)

(B) (P)-(III), (Q)-(IV), (R)-(V), (S)-(I), (T)-(II)

(C) (P)-(IV), (Q)-(III), (R)-(II), (S)-(I), (T)-(V)

(D) (P)-(III), (Q)-(IV), (R)-(I), (S)-(V), (T)-(II)

Key: (A)

20. In a certain month, the reference crop evapotranspiration at a location is 6 mm/day. If the crop coefficient and soil coefficient are 1.2 and 0.8, respectively, the actual evapotranspiration in mm/day is

(A) 5.76 (B) 7.20 (C) 6.80 (D) 8.00

Key: (A)

Sol: Evapotranspiration = 6 mm/day

Crop coefficient $(K_C) = 1.2$

Soil coefficient $(K_S) = 0.8$

Actual evapotranspiration $= K_S \times K_C \times \text{reference evapotranpiration}$
$= 1.2 \times 0.8 \times 6.8 = 5.76 \text{ mm/day}$

21. The dimension of dynamic viscosity is:

(A) $M L^{-1} T^{-1}$ (B) $M L^{-1} T^{-2}$ (C) $M L^{-2}T^{-2}$ (D) $M L^{0}T^{-1}$

Key: (A)

Sol: Unit of dynamic viscosity $= \frac{kg}{m.s} = \frac{M}{L.T} = M.L^{-1}T^{-1}$

22. A process equipment emits 5 kg/h of volatile organic compounds (VOCs). If a hood placed over the process equipment captures 95% of the VOCs, then the fugitive emission in kg/h is

(A) 0.25 (B) 4.75 (C) 2.50 (D) 0.48

Key: (A)

Sol: VOC emission = 5 kg/hr

$\eta = 95\%$

Fugitive emission $= \frac{\text{VOC emission}}{100} \times 5 = 5 \text{ kg/hr} \times \frac{5}{100} = 0.25 \text{ kg/hr}$

23. Match the following attributes of a city with the appropriate scale of measurements.

Attribute	Scale of Measurement
(P) Average temperature (°C) of a city	**(I)** Interval
(Q) Name of a city	**(II)** Ordinal
(R) Population density of a city	**(III)** Nominal
(S) Ranking of a city based on ease of business	**(IV)** Ratio

Which one of the following combinations is correct?

(A) (P)-(I), (Q)-(III), (R)-(IV), (S)-(II)

(B) (P)-(II), (Q)-(I), (R)-(IV), (S)-(III)

(C) (P)-(II), (Q)-(III), (R)-(IV), (S)-(I)

(D) (P)-(I), (Q)-(II), (R)-(III), (S)-(IV)

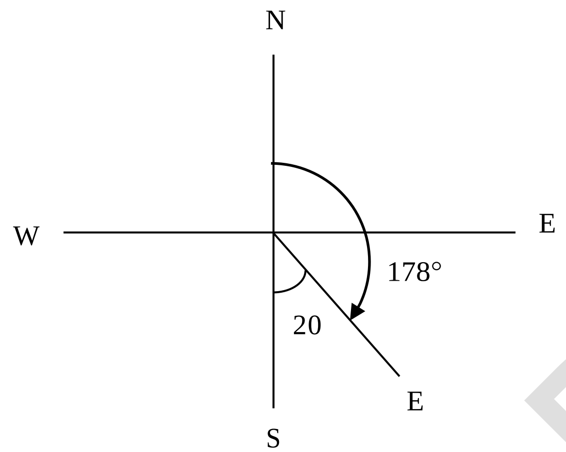
Key: (A)

24. If the magnetic bearing of the Sun at a place at noon is S 2° E, the magnetic declination (in degrees) at that place is

(A) 2° E (B) 2° W (C) 4° E (D) 4° W

Key: (A)

Sol:



Declination = Total bearing – Magnetic bearing = 180° - 178° = 2°E

25. P and Q are two square matrices of the same order. Which of the following statement(s) is/are correct?

(A) If P and Q are invertible, then $[PQ]^{-1} = Q^{-1}P^{-1}$

(B) If P and Q are invertible, then $[QP]^{-1} = P^{-1}Q^{-1}$

(C) If P and Q are invertible, then $[PQ]^{-1} = P^{-1}Q^{-1}$.

(D) If P and Q are not invertible, then $[PQ]^{-1} = Q^{-1}P^{-1}$

Key: (A, B)

Sol: We know that $(AB)^{-1} = B^{-1}A^{-1}$ if A and B are invertible square matrices of same order

∴ If P and Q are invertible, then

$(PQ)^{-1} = Q^{-1}P^{-1}$ and $(QP)^{-1} = P^{-1}Q^{-1}$

If P and Q are not invertible then $(PQ)^{-1} \neq Q^{-1}P^{-1}$

∴ Option A, B

26. In a solid waste handling facility, the moisture contents (MC) of food waste, paper waste, and glass waste were found to be MCf, MCp, and MCg, respectively. Similarly, the energy contents (EC) of plastic waste, food waste, and glass waste were found to be ECpp, ECf, and ECg, respectively. Which of the following statement(s) is/are correct?

(A) MCf > MCp > MCg
(B) ECpp > ECf > ECg
(C) MCf < MCp < MCg
(D) ECpp < ECf < ECg

Key: (A, B)

27. To design an optimum municipal solid waste collection route, which of the following is/are NOT desired:

(A) Collection vehicle should not travel twice down the same street in a day.
(B) Waste collection on congested roads should not occur during rush hours in morning or evening.
(C) Collection should occur in the uphill direction.
(D) The last collection point on a route should be as close as possible to the waste disposal facility.

Key: (C)

28. For a traffic stream, v is the space mean speed, k is the density, q is the flow, v_f is the free flow speed, and k_j is the jam density. Assume that the speed decreases linearly with density.

Which of the following relation(s) is/are correct?

(A) $q = k_j k - \left(\frac{k_j}{v_f}\right)k^2$
(B) $q = v_f k - \left(\frac{v_f}{k_j}\right)k^2$
(C) $q = v_f v - \left(\frac{v_f}{k_j}\right)v^2$
(D) $q = k_j v - \left(\frac{k_j}{v_f}\right)v^2$

Key: (B and D)

Sol: As per Green Shield theorem,

$$V = V_F\left(1 - \frac{K}{K_J}\right)$$

$$\text{Flow }(q) = K.V = K.V_F\left(1 - \frac{K}{K_J}\right)$$

$$q = K.V_F - \frac{K^2 V_F}{K_J}$$

$$q = V_F.K - \left(\frac{V_F}{K_J}\right)K^2 \quad \text{...(1)}$$

If we take $K = K_J\left(1 - \frac{V}{V_F}\right)$

$$q = K.V = K_J\left(1 - \frac{V}{V_F}\right)V$$

$$q = K_J V - \frac{V^2}{V_F}K_J$$

29. The error in measuring the radius of a 5 cm circular rod was 0.2%. If the cross-sectional area of the rod was calculated using this measurement, then the resulting absolute percentage error in the computed area is______. (round off to two decimal places)

Key: **(0.4)**

Sol: radius (r) = 5 cm

error = 0.2% $= \frac{0.2}{100} \times 5\text{cm} = 0.01\,\text{cm}$

Area of circular rod $(A) = \pi r^2$

$e_A = 2\pi r.e_r$

Absolution % error $= \frac{e_A}{A} \times 100 = \frac{2\pi r e_r}{\pi r^2} \times 100 = 2e_r \times 100 = 2 \times 0.01 \times 100 \times 0.4$

30. The components of pure shear strain in a sheared material are given in the matrix form:

$$\varepsilon = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Here, Trace $(\varepsilon) = 0$. Given, P=Trace (ε^8) and $Q = \text{Trace}(\varepsilon^{11})$.

The numerical value of (P + Q) is ________. (in integer)

Key: **(32)**

Sol: The characteristic equation of matrix ε is $\lambda^2 - 2 = 0$

$\Rightarrow \lambda = -\sqrt{2}$ and $\sqrt{2}$ are the eigen values of ε.

By properties of eigen values, eigen value of ε^8 are $\left(-\sqrt{2}\right)^8$ and $\left(\sqrt{2}\right)^8$ i.e., 16, 16 and eigen values of ε^{11} are $(\sqrt{2})^{11}$ and $(\sqrt{2})^{11}$ i.e., $-32\sqrt{2}, 32\sqrt{2}$.

$\therefore \quad P = \text{Trace}\left(\varepsilon^8\right) = 16 + 16 = 32$ and $Q = \text{Trace}\left(\varepsilon^{11}\right) = 32$

Since Trace (A) = Sum of the eigen values of A.

$\therefore$ Numerical value of $(P + Q)$ is 32

31. The inside diameter of a sampler tube is 50 mm. The inside diameter of the cutting edge is kept such that the Inside Clearance Ratio (ICR) is 1.0 % to minimize the friction on the sample as the sampler tube enters into the soil. The inside diameter (in mm) of the cutting edge is _________________.

(round off to two decimal places)

Key: (49.5)

Sol: Inside clearance ratio = 1%

$$\frac{D_3 - D_1}{D_1} \times 100 = 1$$

$$\frac{50 - D_1}{D_1} \times 100 = 1$$

$$(50 - D_1) \times 100 = D_1$$

$$5000 - 100D_1 = D_1$$

$$101\, D_1 = 5000$$

$$D_1 = \frac{5000}{101} = 49.5 \text{ mm}$$

32. A concentrically loaded isolated square footing of size 2 m × 2 m carries a concentrated vertical load of 1000 kN. Considering Boussinesq's theory of stress distribution, the maximum depth (in m) of the pressure bulb corresponding to 10 % of the vertical load intensity will be ___________.

(round off to two decimal places)

Key: (4.37)

Sol: Vertical stress $(\sigma_z) = \frac{3}{2\pi}\frac{Q}{z^2}$

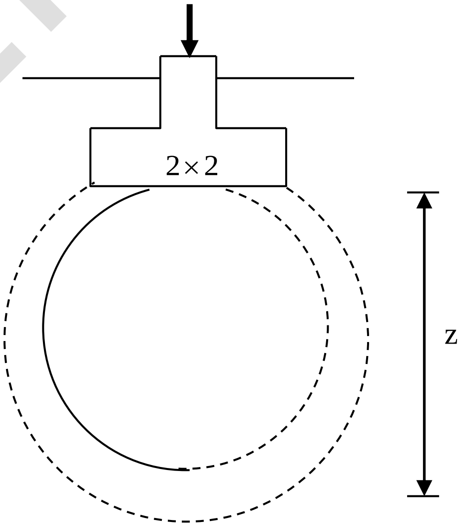
$$V = \frac{Q}{A} = \frac{1000}{2 \times 2} = 250 \text{ kN/m}^2$$

$$\sigma_z = 10\% \text{ of } q = \frac{10}{100} \times 250 = 25 \text{ kN/m}^2$$

$$\sigma_z = \frac{3}{2\pi}\frac{Q}{z^2}$$

$$25 = \frac{3}{2\pi} \times \frac{1000}{z^2}$$

$$z = 4.37 \text{ m}$$



33. In a triaxial unconsolidated undrained (UU) test on a saturated clay sample, the cell pressure was 100 kPa. If the deviatoric stress at failure was 150 kPa, then the undrained shear strength of the soil is _________ kPa. (in integer)

Key: **(75)**

Sol: For VV test

$\phi = 0°$

$\sigma_C = 100\text{ kPa}, \sigma_d = 150\text{ kPa}$

$\sigma_3 = \sigma_c = 100\text{ kPa}$

$\sigma_1 = \sigma_c + \sigma_d = 100 + 150 = 250\text{ kPa}$

$$\text{Undrained shear strength } (C_u) = \frac{\sigma_1 - \sigma_3}{2} = \frac{210 - 100}{2} = 75\text{ kPa}$$

34. A flood control structure having an expected life of n years is designed by considering a flood of return period T years. When T = n, and $n \to \infty$, the structure's hydrologic risk of failure in percentage is ______. (round off to one decimal place)

Key: **(0.632)**

Sol: $\text{Risk failures} = 1 - q^n = 1 - (1 - P)^n = 1 - \left(1 - \frac{1}{T}\right)^n$

$T = n \to \alpha$

$\text{Risk failure} = 1 - \frac{1}{e} = 0.632$

35. The base length of the runway at the mean sea level (MSL) is 1500 m. If the runway is located at an altitude of 300 m above the MSL, the actual length (in m) of the runway to be provided is ____________. (round off to the nearest integer)

Key: **(1605)**

Sol: Base length = 1500 m

Elevation = 300 m

$$\text{Correction for elevation } = \frac{7}{100} \times \frac{300}{300} \times 1500 = 100\text{ m } (+)\text{ve}$$

Actual length or runway = 1500 + 105 = 1605 m

Q. No. 36-65 Carry Two Marks Each

36. Consider the polynomial $f(x) = x^3 - 6x^2 + 11x - 6$ on the domain S given by $1 \le x \le 3$. The first and second derivatives are $f'(x)$ and $f''(x)$.

Consider the following statements:

I. The given polynomial is zero at the boundary points $x = \mathbf{1}$ and $x = \mathbf{3}$.

II. There exists one local maxima of f(x) within the domain S.

III. The second derivative $f''(x) > 0$ throughout the domain S.

IV. There exists one local minima of f(x) within the domain S.

The correct option is:

(A) Only statements I, II and III are correct.

(B) Only statements I, II and IV are correct.

(C) Only statements I and IV are correct.

(D) Only statements II and IV are correct.

Key: (B)

Sol: $f(x) = x^3 - 6x^2 + 11x - 6$ for $1 \le x \le 3$

$\Rightarrow f(1) = 1 - 6 + 11 - 6 = 0$ and $f(3) = 27 - 54 + 33 - 6 = 0$

$\therefore$ Given polynomial f(x) is 0 at x = 1 and 3

$f'(x) = 3x^2 - 12x + 11 = 0$ gives

$$x = \frac{12 \pm \sqrt{144 - 132}}{6} = \frac{12 \pm 2\sqrt{3}}{6} = 2 \pm \frac{1}{\sqrt{3}}$$

$\Rightarrow x = 2 + \frac{1}{\sqrt{3}}, 2 - \frac{1}{\sqrt{3}} \in [1,3] = $ S, are the stationary points

$f''(x) = 6x - 12$

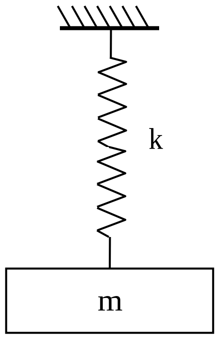
$$\Rightarrow f''\left(2 + \frac{1}{\sqrt{3}}\right) = 6\left(2 + \frac{1}{\sqrt{3}}\right) - 12 = \frac{6}{\sqrt{3}} > 0 \text{ and } f''\left(2 - \frac{1}{\sqrt{3}}\right) = \frac{-6}{\sqrt{3}} < 0$$

$\therefore$ f(x) has local minimum at $x = 2 + \frac{1}{\sqrt{3}}$ and local maximum at $x = 2 - \frac{1}{\sqrt{3}}$

Here $f''(x) \not> 0$ for $x = 2 - \frac{1}{\sqrt{3}} \in [1,3]$ and the domain S is $[1,3]$ i.e., $1 \le x \le 3$

$\therefore$ Statements I, II and IV are correct. Option (B)

37. An undamped spring-mass system with mass m and spring stiffness k is shown in the figure. The natural frequency and natural period of this system are ω rad/s and T s, respectively. If the stiffness of the spring is doubled and the mass is halved, then the natural frequency and the natural period of the modified system, respectively, are



(A) 2 ω rad/s and T/2s (B) ω /2 rad/s and 2Ts

(C) 4 ω rad/s and T/4s (D) ω rad/s and Ts

Key: (A)

Sol: The natural frequency of given vibrating system is

$$\omega_n = \sqrt{\frac{k}{m}} = \omega$$

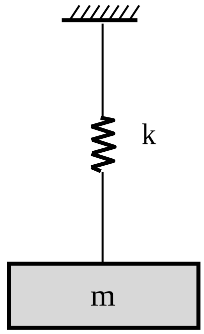
Time period $T = \frac{2\pi}{\omega_n} = \frac{2\pi}{\omega}$

Given that stiffness is doubled, $k' = 2k$

Maxx is halved, $m' = \frac{m}{2}$

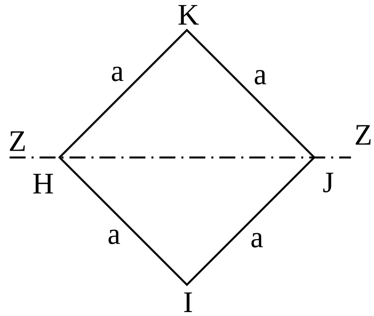
$$\omega_n' = \sqrt{\frac{k'}{m'}} = \sqrt{\frac{2k}{\frac{m}{2}}} = 2\sqrt{\frac{k}{m}} = 2\omega$$

$$T' = \frac{2\pi}{\omega_n'} = \frac{2\pi}{2\omega} = \frac{T}{2}$$



∴ Option ‘A’ is correct.

38. For the square steel beam cross-section shown in the figure, the shape factor about $z-z$ axis is S and the plastic moment capacity is M_P. Consider yield stress f_y = 250 MPa and a = 100 mm.



The values of S and M_P (rounded-off to one decimal place) are

(A) S = 2.0, M_P = 58.9 kN-m (B) S = 2.0, M_P = 100.2 kN-m

(C) S = 1.5, M_P= 58.9 kN-m (D) S = 1.5, M_P = 100.2 kN-m

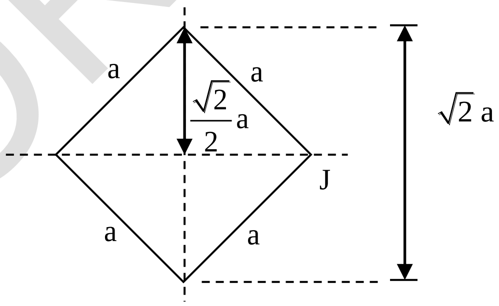
Key: **(A)**

Sol: Shape factor for diameter shape = 2

$$\text{Shaper factor} = \frac{\text{Plastic moment}}{\text{Elastic moment}} = \frac{M_p}{M_y}$$

$$S = \frac{f_y.Z_P}{f_y.Z_e}$$

$$M_P = S \times f_y \times Z_e = 2 \times 250 \times \frac{\sqrt{2} \times 100^3}{12} \times 10^{-6} = 58.93 \text{ kNm}$$

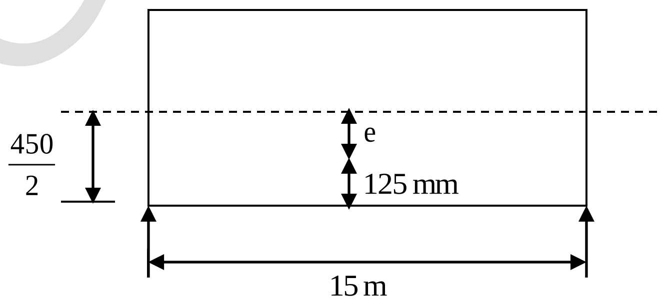


39. A post-tensioned concrete member of span 15 m and cross-section of 450mm×450mm is prestressed with three steel tendons, each of cross-sectional area 200mm². The tendons are tensioned one after another to a stress of 1500 MPa. All the tendons are straight and located at 125 mm from the bottom of the member. Assume the prestress to be the same in all tendons and the modular ratio to be 6. The average loss of prestress, due to elastic deformation of concrete, considering all three tendons is

(A) 14.16 MPa (B) 7.08 MPa (C) 28.32 MPa (D) 42.48 MPa

Key: **(A)**

Sol:



Eccentricity $(e) = \frac{450}{2} - 125 = 225 - 125 = 100 \text{ mm}$

Stress in concrete at the level of tendon

$$f_e = \frac{P}{A} + \frac{P.e}{I}.e = \frac{300 \times 10^3}{450 \times 450} + \frac{300 \times 10^3 \times 100}{\left(\frac{450^4}{12}\right)} \times 100 = 1.48 + 0.88 = 2.36 \text{ MPa}$$

Loss of stress due to elastic deformation after tensioning first wire = 0.

Loss of stress due to elastic deformation $= mf_e = 6 \times 2.36 = 14.16 \text{ MPa}$ after tensioning 2nd wire.

Loss of stress due to elastic deformation $= mf_e = 6 \times 2.36 = 14.16 \text{ MPa}$ after tensioning 3rd wire.

Total loss in tendon (3) = 2 × 14.16 = 28.32

Total loss in tendon (2) = 14.16 MPa

Total loss in tendon (1) = 0

Average loss $= \frac{28.32 + 14.16 + 0}{3} = 14.16 \text{ MPa}$

40. Match the following in Column X with Column Y:

Column X	Column Y
(P) In a triaxial compression test, with increase of axial strain in loose sand under drained shear condition, the volumetric strain	**(I)** decreases.
(Q) In a triaxial compression test, with increase of axial strain in loose sand under undrained shear condition, the excess pore water pressure	**(II)** increases.
(R) In a triaxial compression test, the pore pressure parameter "B" for a saturated soil	**(III)** remains same.
(S) For shallow strip footing in pure saturated clay, Terzaghi's bearing capacity factor (N_q) due to surcharge	**(IV)** is always 0.0.
	(V) is always 1.0.
	(VI) is always 0.5.

Which one of the following combinations is correct?

(A) (P)-(I), (Q)-(II), (R)-(V), (S)-(V)

(B) (P)-(II), (Q)-(I), (R)-(IV), (S)-(V)

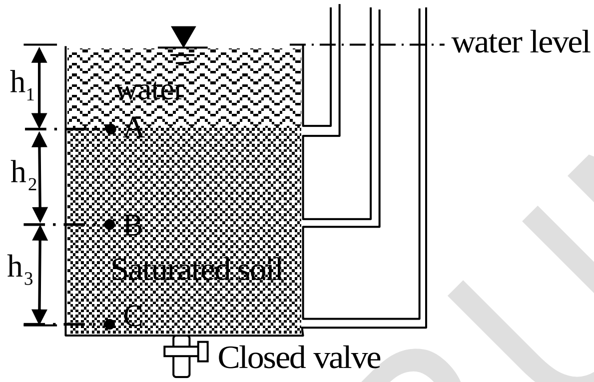
(C) (P)-(I), (Q)-(III), (R)-(VI), (S)-(IV)

(D) (P)-(I), (Q)-(II), (R)-(V), (S)-(VI)

Key: (A)

41. A soil sample is underlying a water column of height h_1, as shown in the figure.

The vertical effective stresses at points A, B, and C are σ'_A and σ'_C, respectively. Let γ_{sat} and γ' be the saturated and submerged unit weights of the soil sample, respectively, and γ_w be the unit weight of water. Which one of the following expressions correctly represents the sum $(\sigma'_A + \sigma'_B + \sigma'_C)$?



(A) $(2h_2 + h_3)\gamma'$ (B) $(h_1 + h_2 + h_3)\gamma'$

(C) $(h_2 + h_3)(\gamma_{sat} - \gamma_w)$ (D) $(h_1 + h_2 + h_3)\gamma_{sat}$

Key: (A)

Sol:

$\bar{\sigma}_A = 0$

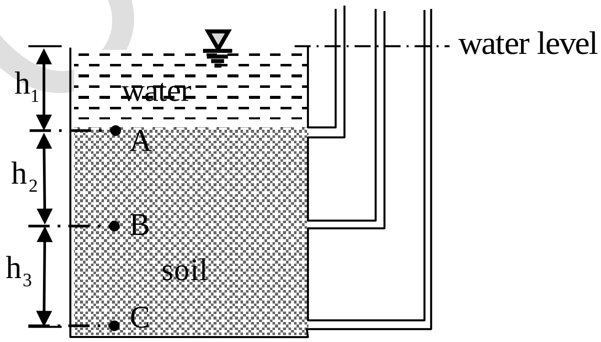
$\bar{\sigma}_B = h_2\gamma_{sub}$

$\bar{\sigma}_C = (h_2 + h_3)\gamma_{sub}$

$\bar{\sigma}_A + \bar{\sigma}_B + \bar{\sigma}_c = 0 + h_2\gamma_{sub} + (h_2 + h_3)\gamma_{sub}$

$= (2h_2 + h_3)\gamma_{sub}$

$= (2h_2 + h_3)\gamma'$



42. A 100 mg of HNO_3 (strong acid) is added to water, bringing the final volume to 1.0 liter. Consider the atomic weights of H, N, and O, as 1 g/mol, 14 g/mol, and 16 g/mol, respectively. The final pH of this water is (Ignore the dissociation of water.)

(A) 2.8 (B) 6.5 (C) 3.8 (D) 8.5

Key: (A)

Sol: $HNO_3 \rightarrow H^4 + NO_3^-$

$1 \text{ mole } HNO_3 \rightarrow 1 \text{ mole of } H^+$

$63 \text{ gm of } HNO_3 \rightarrow 1 \text{ gm. of } H^+$

$100 \text{ mg of} HNO_3 \rightarrow ?$

$\frac{1}{63}\times 100 = 1.587.\text{ mg of }H^{+} = 1.587\times 10^{-3}\text{g of }H^{+}$

$p^{H} = -\log_{10}\left[H^{+}\right] = -\log_{10}\left(1.587\times 10^{-3}\right)$

$p^{H} = 2.8$

43. In a city, the chemical formula of biodegradable fraction of municipal solid waste (MSW) is $C_{100}H_{250}O_{80}N$. The waste has to be treated by forced-aeration composting process for which air requirement has to be estimated.

Assume oxygen in air (by weight) = 23 %, and density of air = 1.3 kg/m^3.

Atomic mass: C = 12, H = 1, O = 16, N = 14.

C and H are oxidized completely whereas N is converted only into NH_3 during oxidation.

For oxidative degradation of 1 tonne of the waste, the required theoretical volume of air (in m^3/tonne) will be (round off to the nearest integer)

(A) 4749 (B) 8025 (C) 1418 (D) 1092

Key: **(A)**

Sol: Chemical formula of municipal solid waste is $C_{100}H_{250}O_{80}N$

Given that, C and H are completely oxidized where as 'N' is converted into NH_3

$\therefore \quad C_{100}H_{250}O_{80}N + AO_2 \rightarrow BCO_2 + CH_2O + NH_3$

Given that atomic mass of

C = 12, H = 1, O = 16, N = 14

Balancing 'C' on both sides

$100\times 12 = B\times 12$

$\boxed{\Rightarrow B = 100}$

Balancing 'H' on both sides

$250 \times 1 = C \times 2 \times 1 + 3$

$C = 123.5$

Balancing 'O' on both sides

$80\times 16 + 2\times A\times 16 = B\times 2\times 16 + C\times 16$

$1280 + 32A = 32\times 100 + 123.5\times 16$

$A = 121.75$

$\therefore$ Chemical reaction will becomes as follows,

$C_{100}H_{250}O_{80}N + 121.75O_2 \rightarrow 100CO_2 + 123.5H_2O + NH_3$

2744 gm/mole, 3896 gm/mol

From the above we can say that, for complete combustion of solid waste of 2744 gm/mole requires 3896gm/mole of oxygen.

$\therefore$ For 1000 kgs of solid waste, the oxygen required (1 ton) is

$$=\left(\frac{3896}{2744}\right)\times 1000 = 1419.825\text{ kg}$$

Given that density of air $(\rho) = 1.3\text{ kg/m}^3$

Then density of oxygen will be $= 0.23\times 1.3 = 0.299\text{ kg/m}^3$

Let V = Volume of air occupied (or) volume of oxygen occupied since both are in same sample

Then density of air $= \frac{\text{mass of air}}{V}$

$$1.3 = \frac{\text{mass of air}}{V} \quad \ldots(1)$$

Density of oxygen $= \frac{\text{mass of oxygen}}{V}$

$$0.299 = \frac{1419.825}{V} \quad \ldots(2)$$

Substituting equation (1) and (2)

$$\Rightarrow \frac{1.3}{0.299} = \frac{\text{Mass of air}}{1419.825}$$

Mass of air required $= \left(\frac{1.3}{0.299}\right)(1419.825) = 6173.152\text{ kg}$

Volume of air required $(V) = \frac{\text{Mass of air required}}{\text{Density of air}} = \frac{6173.152}{1.3} = 4748.58\text{ m}^3$

So answer is option 'A'.

44. A single-lane highway has a traffic density of 40 vehicles/km. If the time-mean speed and space-mean speed are 40 kmph and 30 kmph, respectively, the average headway (in seconds) between the vehicles is

(A) 3.00 (B) 2.25 (C) 8.33×10^{-4} (D) 6.25×10^{-4}

Key: **(A)**

Sol: Traffic density (k) = 40 veh/km

Time mean speed = 4- km/hr

Space mean speed $(V_S) = 30\text{ km/hr}$

Flow $= K\times V_S$

$q = KV = 40 \times 30 = 1200\ \text{veh/hr}$

We know that

$$q = \frac{3600}{\text{Time headway}}$$

$$\text{Time headway } \bar{\sigma} = \frac{3600}{q} = \frac{3600}{1200} = 3$$

45. Let $\boldsymbol{y}$ be a non-zero vector of size 2022 × 1. Which of the following statement(s) is/are TRUE?

(A) yy^T is a symmetric matrix.

(B) y^Ty is an eigenvalue of yy^T

(C) yy^T has a rank of 2022.

(D) yy^T is invertible.

Key: **(A, B)**

Sol: Suppose $y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ be a non-zero vector of size 2 × 1

$\Rightarrow yy^T = \begin{bmatrix} 1 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ is a symmetric matrix of order 2.

$y^Ty = \begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = [2]$

Since $\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1 - 1 = 0$ and rank of matrix $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ is 1

$\therefore$ yy^T is singular matrix $\Rightarrow yy^T$ is not invertible.

$\Rightarrow$ yy^T has a rank of 1 but not 2.

The eigen value of y^Ty is 2.

The characteristic equations of yy^T is $\begin{vmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} = 0$

$\Rightarrow (1-\lambda)^2 - 1 = 0 \Rightarrow \lambda^2 - 2\lambda = 0 \Rightarrow \lambda = 0, 2$ are the eigen values of yy^T

$\therefore$ y^Ty is an eigen value of yy^T

$\therefore$ (A), (B) are true.

Similarly (A), (B) are true, if y be a non-zero vector of size 2022 × 1

46. Which of the following statement(s) is/are correct?

(A) If a linearly elastic structure is subjected to a set of loads, the partial derivative of the total strain energy with respect to the deflection at any point is equal to the load applied at that point.

(B) If a linearly elastic structure is subjected to a set of loads, the partial derivative of the total strain energy with respect to the load at any point is equal to the deflection at that point.

(C) If a structure is acted upon by two force system P_a and P_b, in equilibrium separately, the external virtual work done by a system of forces P_b during the deformations caused by another system of forces P_a is equal to the external virtual work done by the Pa system during the deformation caused by the P_b system.

(D) The shear force in a conjugate beam loaded by the M/EI diagram of the real beam is equal to the corresponding deflection of the real beam.

Key: (A, B, C)

47. Water is flowing in a horizontal, frictionless, rectangular channel. A smooth hump is built on the channel floor at a section and its height is gradually increased to reach choked condition in the channel. The depth of water at this section is y_2 and that at its upstream section is y_1. The correct statement(s) for the choked and unchoked conditions in the channel is/are

(A) In choked condition, y_1 decreases if the flow is supercritical and increases if the flow is subcritical.

(B) In choked condition, y_2 is equal to the critical depth if the flow is supercritical or subcritical.

(C) In unchoked condition, y_1 remains unaffected when the flow is supercritical or subcritical.

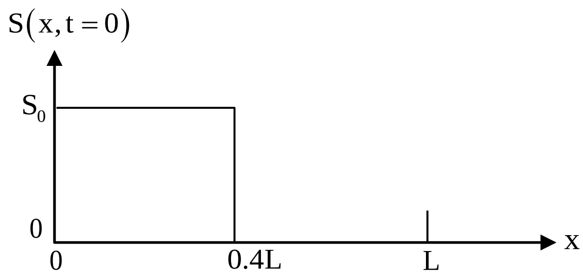
(D) In choked condition, y_1 increases if the flow is supercritical and decreases if the flow is subcritical.

Key; (A, B, C)

48. The concentration s(x,t) of pollutants in a one-dimensional reservoir at position x and time t satisfies the diffusion equation

$$\frac{\partial s(x,t)}{\partial t} = D\frac{\partial^2 s(x,t)}{\partial x^2}$$

on the domain $0 \le x \le L$, where D is the diffusion coefficient of the pollutants. The initial condition s(x, 0) is defined by the step-function shown in the figure.

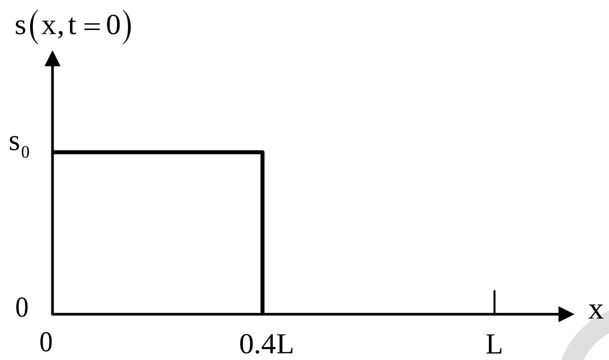


The boundary conditions of the problem are given by $\frac{\partial s(x,t)}{\partial x}=0$ at the boundary points x=0 and x=L at all times. Consider D=0.1m^2/s, $s_0=5\mu mol/m$, and L=10m.

The steady state concentration $\tilde{S}\left(\frac{L}{2}\right)=S\left(\frac{L}{2},\infty\right)$ at the center $x=\frac{L}{2}$ the reservoir $(\text{in } \mu mol/m)$ is _________. (in integer)

Key: (2)

Sol:



From figure $s(x,t)$

At $x=s(0,1)=s_0=5$

At $x=0.4L\Rightarrow s(0.4L,t)=s_0=5$

At $x=L\Rightarrow s(L,t)=0$

From $x=0$ to $x=0.4L$

Concentration of pollutant

$=5\,\mu\,mol/m\times(0.4\times10m)=20\,\mu\,mol$

$x=0.4L$ to $x=L$

Concentration of pollutant = 0

Total concentration of pollutant in 10 m $=20\,\mu\,mol$

In infinite time this concentration will be diluted

So concentration of pollutant per m

$=\frac{20}{10}\mu\,mol/m=2\mu\,mol/m$

Under steady state condition, concentration of pollutant will be uniformly distributed.

Steady state concentration at $x=\frac{L}{2}=2\,\mu\,mol/m$

49. A pair of six-faced dice is rolled thrice. The probability that the sum of the outcomes in each roll equals 4 in exactly two of the three attempts is ______. (round off to three decimal places)

Key: **(0.019)**

Sol: Event sum 4 = $\{(1,3),(2,2),(3,1)\}$ and sample space S has 36 outcomes.

Let $P = P_r$ (sum 4 in any roll of pair of dice) $= \frac{3}{36} = \frac{1}{12}$

$$q = 1 - p = \frac{11}{12}$$

Let X be a Binomial random variable denote number of times that sum 4 is obtained

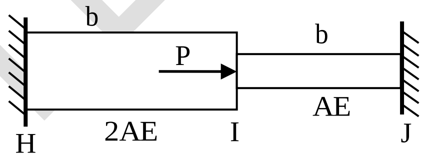
$\Rightarrow X = 0,1,2,3$ and $n = 3 \rightarrow$ thrice

then $P_r(X = x) = n_{c_x} . p^x q^{n-x}$

$\therefore$ Probability that the sum 4 obtains exactly two times in three attempts is

$$P_r(x = 2) = 3C_2 \left(\frac{1}{12}\right)^2 \left(\frac{11}{12}\right) = \frac{11}{576} \approx 0.019$$

50. Consider two linearly elastic rods HI and IJ, each of length b, as shown in the figure. The rods are co-linear, and confined between two fixed supports at H and J. Both the rods are initially stress free. The coefficient of linear thermal expansion is α for both the rods. The temperature of the rod IJ is raised by ΔT, whereas the temperature of rod HI remains unchanged. An external horizontal force P is now applied at node I. It is given that $\alpha = 10^{-6} °C^{-1}$, $\Delta T = 50°C$, b=2m, AE=10^6N. The axial rigidities of the rods HI and IJ are 2AE and AE, respectively.



To make the axial force in rod HI equal to zero, the value of the external force P (in N) is _________. (round off to the nearest integer)

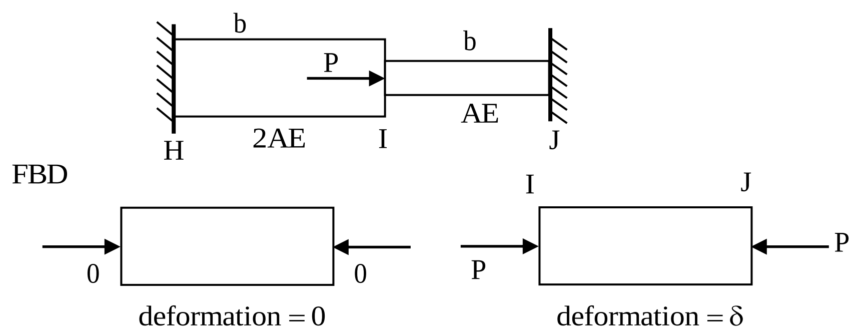
Key: **(50)**

Sol: $\delta = (\delta)_{\text{axial load}} + \delta_{\text{temp}} = 0$

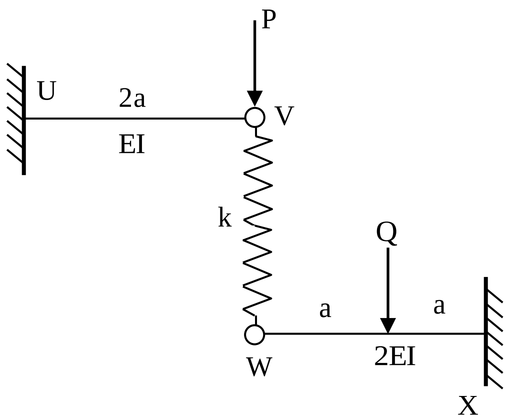
$$\frac{Pb}{AE} - b\alpha\Delta T = 0$$

$$\frac{Pb}{AE} = b\alpha\Delta T$$

$$P = AE\alpha\Delta T = 10^6 \times 10^{-6} \times 50 = 50 \text{ N}$$



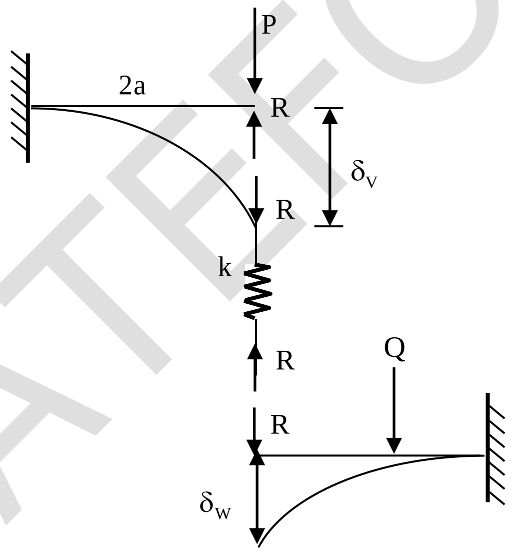
51. The linearly elastic planar structure shown in the figure is acted upon by two vertical concentrated forces. The horizontal beams UV and WX are connected with the help of the vertical linear spring with spring constant k = 20kN/m. The fixed supports are provided at U and X. It is given that flexural rigidity $EI = 10^5 kN-m^2$, $P = 100kN$, and a=5m. Force Q is applied at the center of beam WX such that the force in the spring VW becomes zero.



The magnitude of force Q (in kN) is ________. (round off to the nearest integer)

Key: **(640)**

Sol:



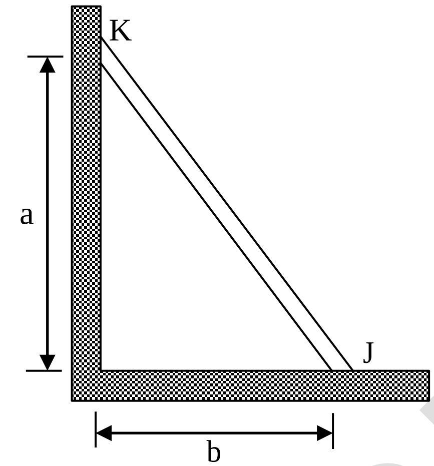
$$\delta_V = \delta_W$$

$$\frac{P(2a)^3}{3EI} = \frac{Q.(a)^3}{3(2EI)} + \frac{Q.a^2}{2(2EI)} + a$$

$$\frac{8}{3}P = \left(\frac{1}{6} - \frac{1}{4}\right)Q$$

$$Q = \frac{32}{5}P = \frac{32}{5} \times 100 = 640\ kN$$

52. A uniform rod KJ of weight w shown in the figure rests against a frictionless vertical wall at the point K and a rough horizontal surface at point J. It is given that w = 10 kN, a = 4 m and b = 3 m.



The minimum coefficient of static friction that is required at the point J to hold the rod in equilibrium is ___________. (round off to three decimal places)

Key: (0.375)

Sol: From equilibrium equations

$\Sigma V_V = 0$

$R_J = 10\,kN$

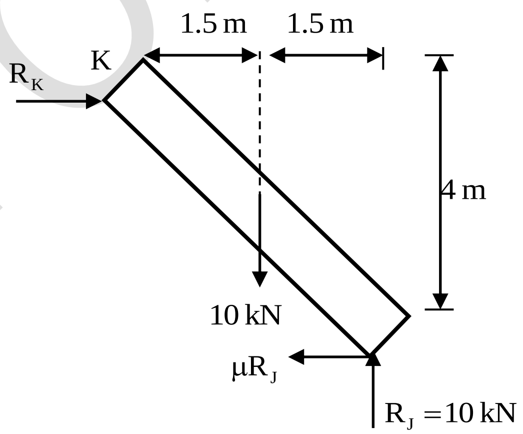
Taking moment about k

$\Sigma M_K = 0$

$R_J \times 3 - 10 \times 1.5 - \mu R_J \times 4 = 0$

$3R_J = 15 + q\mu R_J$

$\mu = \frac{15 - 3R_J}{4R_J} = \frac{15 - 3\times 10}{4\times 10} = \frac{15}{40} = 0.375$



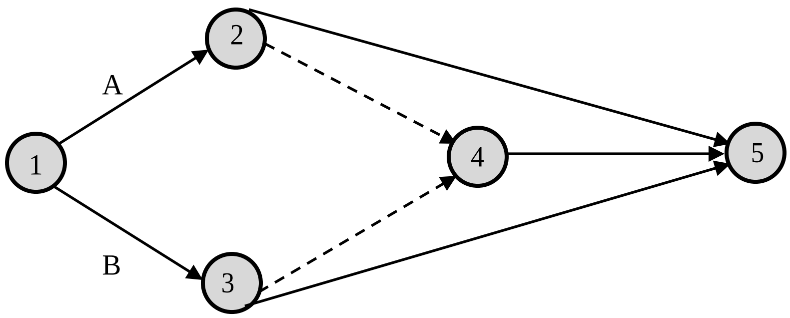
53. The activities of a project are given in the following table along with their durations and dependency.

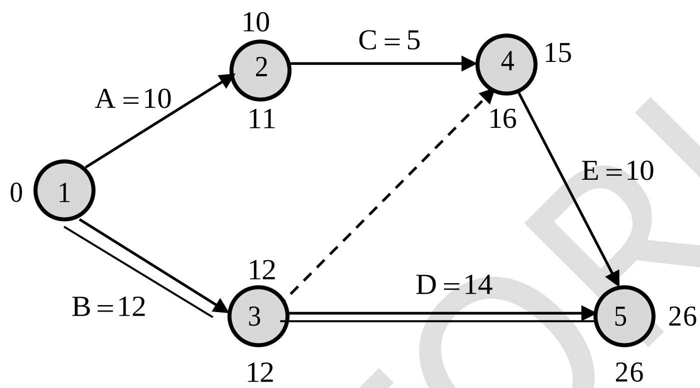
Activities	Duration (days)	Depends on
A	10	-
B	12	-
C	5	A
D	14	B
E	10	B,C

The total float of the activity E (in days) is ________. (in integer)

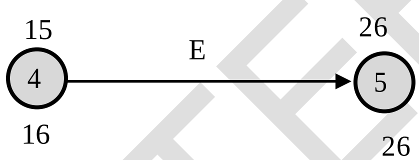
Key: **(1)**

Sol:





For activity E



Total float $=(T_E - T_j) - t = (26-15) - 10 = 11 - 10 = 1$

54. A group of total 16 piles are arranged in a square grid format. The center-tocenter spacing (s) between adjacent piles is 3m. The diameter (d) and length of embedment of each pile are 1m and 20 m, respectively. The design capacity of each pile is 1000 kN in the vertical downward direction. The pile group efficiency (η_g) is given by $\eta_g = 1 - \frac{\theta}{90}\left[\frac{(n-1)m + (m-1)n}{mn}\right]$

Where m and n are number of rows and columns in the plan grid of pile arrangement, and $\theta = \tan^{-1}\left(\frac{d}{s}\right)$.

The design value of the pile group capacity (in kN) in the vertical downward direction is ___________________. (round off to the nearest integer)

Key: **(11088)**

Sol: 16 Piles-4 × 4 Square format

S = 3 m; d = 1m; L = 20 m

$Q_i = 1000 \text{ kN}$

$Q_g = ?$

$$n_g = \frac{Q_g}{N.Q_i}$$

$$n_g = 1 - \frac{\theta}{90°}\left[\frac{m(n-1)+n(m-1)}{mn}\right]$$

$m = 4; n = 4$

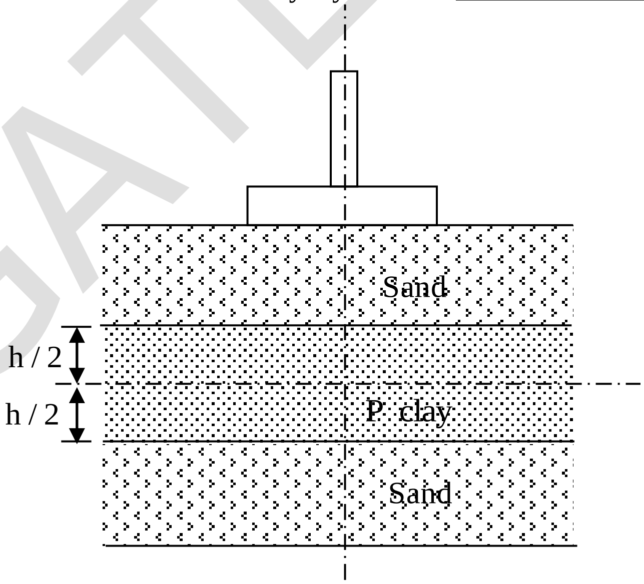
$$\theta = \tan^{-1}\left(\frac{1}{3}\right) = 18.43°$$

$$n_g = 1 - \frac{18.43}{90}\left[\frac{4\times 3+3\times 4}{4\times 4}\right]$$

$n_g = 0.693 = 69.3\%$

$Q_g = n_g.N.Q_i = 0.693\times 16\times 1000 = 11{,}088 \text{ kN}$

55. A saturated compressible clay layer of thickness h is sandwiched between two sand layers, as shown in the figure. Initially, the total vertical stress and pore water pressure at point P, which is located at the mid-depth of the clay layer, were 150 kPa and 25 kPa, respectively. Construction of a building caused an additional total vertical stress of 100 kPa at P. When the vertical effective stress at P is 175 kPa, the percentage of consolidation in the clay layer at P is ____________. (in integer)



Key: (57% or 50%)

Sol: Before construction of building:

At point P, $\sigma = 150\text{ kPa}$

$u = 25\text{ kPa}$

$\sigma' = \sigma - u = 125\text{ kPa}$

After construction of building:

Increase in total stress, $\Delta\sigma = 100\text{ kPa}$

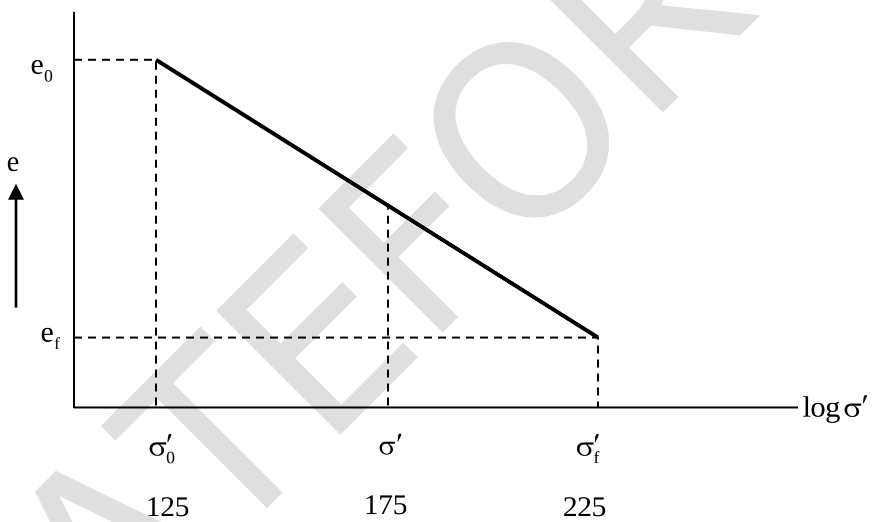
Due to increase in the stress, the soil undergoes consolidation.

Consolidation:

At beginning of consolidation, $\sigma'_0 = 125\text{ kPa}$

At end of consolidation, $\sigma'_f = \sigma'_0 + \Delta\sigma = 125 + 100 = 225\text{ kPa}$

At some intermediate stage of consolidation, the given $\sigma' = 175\text{ kPa}$



Final consolidation settlement, $S_f = H\dfrac{C_c}{1+e_0}.\log_{10}\dfrac{\sigma'_f}{\sigma'_0}$

$$S_f \alpha \log_{10}\frac{\sigma'_f}{\sigma'_0}$$

Settlement that would occur when σ' reaches to 175 kPa

$$S = H\frac{C_C}{1+e_0}.\log_{10}\frac{\sigma'}{\sigma'_0}$$

$$S \alpha \log_{10}\frac{\sigma'}{\sigma_0^1}$$

The degree of consolidation, $U = \frac{S}{S_f} \times 100$

$$U = \frac{\log_{10} \frac{\sigma'}{\sigma'_0}}{\log_{10} \frac{\sigma'_f}{\sigma'_0}} = \frac{\log_{10} \frac{175}{125}}{\log_{10} \frac{225}{125}} = 57.24\%$$

However, if settlement is assumed proportional to σ' then, $U = \frac{\bar{u} - \bar{u}}{\bar{u}_1} \times 100$

$\bar{u}_1 =$ initial excess pore pressure $= \Delta\sigma = 100\,\text{kPa}$

$\bar{u} =$ excess pore pressure at some intermediate stage of consolidation

$= 225 - 175 = 50\,\text{kPa}$

$$\therefore U = \frac{100 - 50}{100} \times 100 = 50\%\,(\text{Ans})$$

The above two solutions give two different answers. Since settlement (s) or void ratio (e) is actually proportional to log σ', the first solution may be more appropriate.

56. A hydraulic jump takes place in a 6 m wide rectangular channel at a point where the upstream depth is 0.5 m (just before the jump). If the discharge in the channel is 30m^3/s and the energy loss in the jump is 1.6 m, then the Froude number computed at the end of the jump is ___________. (round off to two decimal places)

(Consider the acceleration due to gravity as 10 m/s^2.)

Key: (0.32)

Sol: $Fr_1 = \frac{q}{\sqrt{g.y_1^{\frac{3}{2}}}} = \frac{5}{\sqrt{10} \times 0.5^{\frac{3}{2}}}$

$Fr_1 = 4.47$

$$\therefore \frac{Fr_2}{Fr_1} = \left(\frac{y_1}{y_2}\right)^{\frac{3}{2}}$$

$$\frac{Fr_2}{4.47} = \left(\frac{0.5}{2.92}\right)^{\frac{3}{2}}$$

$Fr_2 = 0.316 \approx 0.32$

57. A pump with an efficiency of 80% is used to draw groundwater from a well for irrigating a flat field of area 108 hectares. The base period and delta for paddy crop on this field are 120 days and 144 cm, respectively. Water application efficiency in the field is 80%. The lowest level of water in the well is 10m below the ground. The minimum required horse power (h.p.) of the pump is ________. (round off to two decimal places). (Consider 1 h.p. = 746 W; unit weight of water = 9810 N/m^3)

Key: **(30.82)**

Sol: Given pump efficiency = 80%

Field area = 108 hectares

Base period of crop (B) = 120 days

Delta of crop $(\Delta) = 144\text{ cm}$

Water application efficiency $(\eta_a) = 80\%$

GWT depth from ground = 10 m (below)

1 h.p = 746 W

$\gamma_w = 9810\ \text{N/m}^3$

Duty of crop $(D) = \dfrac{864 \times B}{\Delta} = \dfrac{864 \times 120}{144}$

D = 720 hac/cumecs

Discharge required $(Q) = \dfrac{\text{Area}}{\text{Duty}} = \dfrac{108}{720} = 0.15\ \text{m}^3/\text{sec}$

Since $(\eta_a) = 80\%$

$$Q_{design} = \frac{Q}{\eta_a} = \frac{0.15}{0.8}$$

$$Q_{design} = 0.1875\ \text{m}^3/\text{sec}$$

Minimum power required $= (\rho g\, Q_{design} \times H)/\eta_{pump}$

$$P = \frac{9810 \times 0.1875 \times 10}{746} \times \frac{1}{0.8} = (30.82\text{ HP})$$

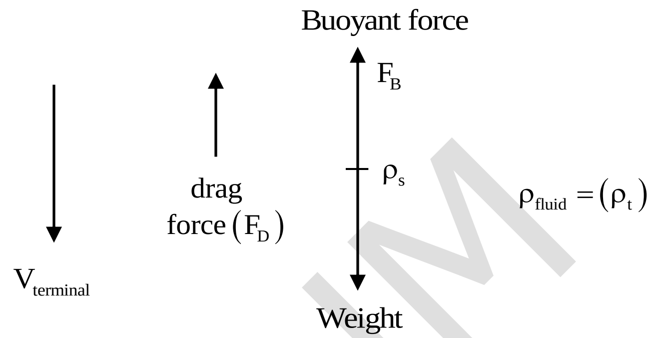
58. Two discrete spherical particles (P and Q) of equal mass density are independently released in water. Particle P and particle Q have diameters of 0.5 mm and 1.0 mm, respectively. Assume Stokes' law is valid. The drag force on particle Q will be________ times the drag force on particle P.

(round off to the nearest integer)

Key: **(8)**

Sol: In case of discrete particle settling and Stoke's law valid, at terminal velocity, since there is no change in velocity, the net force on the body is zero. Hence,

(Weight of sphere –buoyant force) = Drag force (F_D)



$$\Rightarrow F_D = \left(\frac{\pi}{6}D^3\rho_S g - \frac{\pi}{6}D^3\rho_f g\right)$$

$$\boxed{F_D = \frac{\pi}{6}gD^3(\rho_S - \rho_f)}$$

For density of medium (ρ_f) and mass density of sphere (ρ_S) constant,

Drag force $(F_D) \propto D^3$

For particle P, diameter $(D_P) = 0.5\text{ mm}$

For particle Q, diameter $(D_Q) = 1\text{ mm}$

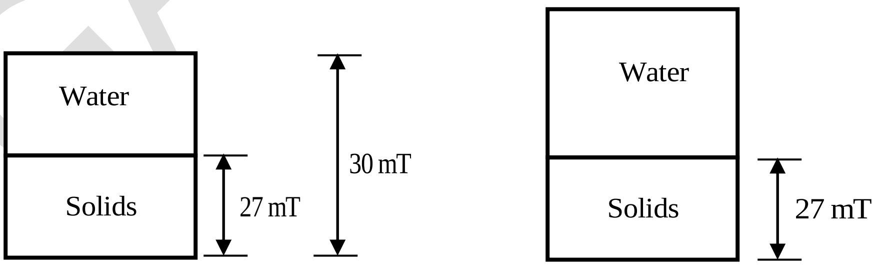
$$\Rightarrow \frac{(F_D)_Q}{(F_D)_P} = \frac{(D_Q)^3}{(D_P)^3} = \left(\frac{1}{0.5}\right)^3 = 8$$

$$\Rightarrow (F_D)_Q = 8 \times (F_D)_P$$

59. At a municipal waste handling facility, 30 metric ton mixture of food waste, yard waste, and paper waste was available. The moisture content of this mixture was found to be 10%. The ideal moisture content for composting this mixture is 50%. The amount of water to be added to this mixture to bring its moisture content to the ideal condition is _______metric ton. (in integer)

Key: **(24)**

Sol:



Weight of water initially present $= \frac{10}{100} \times 30 = 3\text{ mT}$

Weight of solids = 30 – 3 = 27 mT

In ideal condition,

Weight of solids = 27 mT

M/C = 500%

Weight of water = 27 mT

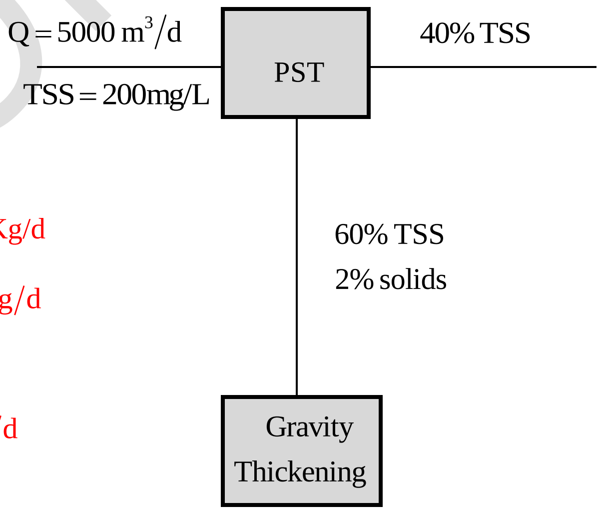
Amount of water needed to get ideal condition = 27 – 3 = 24 mT

60. A sewage treatment plant receives sewage at a flow rate of 5000 m^3/day. The total suspended solids (TSS) concentration in the sewage at the inlet of primary clarifier is 200 mg/L. After the primary treatment, the TSS concentration in sewage is reduced by 60 %. The sludge from the primary clarifier contains 2 % solids concentration. Subsequently, the sludge is subjected to gravity thickening process to achieve a solids concentration of 6 %. Assume that the density of sludge, before and after thickening, is 1000 kg/m^3.

The daily volume of the thickened sludge (in m^3/day) will be_________. (round off to the nearest integer)

Key: (10)

Sol: Weight of TSS at inlet of PST



$$= 5000 \times 10^3 \frac{L}{d} \times 200 \times 10^{-6} \frac{Kg}{L}$$

$$= 1000\ Kg/d$$

Weight of solids in sludge from PST = 0.6 × 1000 = 600 Kg/d

Weight of sludge before thickness $= \dfrac{600}{\frac{2}{100}} Kg/d = 30000\ Kg/d$

Weight of sludge after thickening $= \dfrac{600}{\frac{6}{100}} Kg/d = 1000\ Kg/d$

Daily volume of thickness sludge $= \dfrac{10000}{1000} m^2 = 10\ m^3 / day$

61. A sample of air analyzed at 25 °C and 1 atm pressure is reported to contain 0.04 ppm of SO_2. Atomic mass of S = 32, O = 16.

The equivalent SO_2 concentration (in µg/m^3) will be__________. (round off to the nearest integer)

Key: (105)

Sol: Concentration of SO_2 in ppm = 0.04

Let's equivalent concentration in $\mu g/m^3$ is x.

$\Rightarrow x\mu g$ of SO_2 present in 1 m^3 of air.

We know, 1 mole of SO_2 (at 0°C, 1 atm) has volume of 22.4L.

at 25°C and 1 atm, volume of SO_2 is

$$=\frac{22.4}{273+0}\times(273+25)=24.45 \text{ lit}$$

$\frac{x\times10^{-6}}{64}$ mole of SO_2 has volume

$$=\frac{24.45\times x\times10^{-6}}{64} \text{lit. in } 1\text{ m}^3 \text{ of air}$$

$0.382x\times10^{-3}m^3$ of SO_2 present in $10^6 m^3$ of air.

$$\Rightarrow 0.382x\times10^{-3}=0.04$$
$$x=105$$

0.04 ppm of $SO_2 = 105\mu g/m^3$ of SO_2

62. A parabolic vertical crest curve connects two road segments with grades +1.0% and -2.0%. If a 200 m stopping sight distance is needed for a driver at a height of 1.2 m to avoid an obstacle of height 0.15 m, then the minimum curve length should be ______ m. (round off to the nearest integer)

Key: **(272.91)**

Sol: Given that, $n_1=+1\%$ and $n_2=-2\%$

$$n=n_1-n_2=1-(-2)=3\%$$

SSD = 220 m

and $h_1=1.2$ m and $h_2=0.15$ m

As given n_1 up gradient, and n_2 – down gradient

So curve is summit curve

Assume L > SSD

$$L=\frac{NS^2}{2\left(\sqrt{h_1}+\sqrt{h_2}\right)^2}=\frac{3}{100}\times\frac{(200)^2}{2\times\left(\sqrt{1.2}+\sqrt{0.15}\right)^2}$$
$$L=272.91\text{ m}>200\text{ m}$$ \
$$L=272.91\text{ m}$$

63. Assuming that traffic on a highway obeys the Greenshields model, the speed of a shockwave between two traffic streams (P) and (Q) as shown in the schematic is _______ kmph. (in integer)

Direction of Traffic →

(P) Flow=1200 vehicles/hour Speed=60kmph	(Q) Flow=1800 Vehicles/hour Speed=30kmph

Key: **(15)**

Sol: Flow = Speed × Density

$$\text{Density} = \frac{\text{Flow}}{\text{Speed}} = \frac{q_Q - q_P}{\frac{q_Q}{V_Q} - \frac{q_P}{V_P}} = \frac{1800-1200}{\frac{1800}{30} - \frac{1200}{60}} = \frac{600}{60-20} = \frac{600}{40} = 15\text{ Kmph}$$

64. It is given that an aggregate mix has 260 grams of coarse aggregates and 240 grams of fine aggregates. The specific gravities of the coarse and fine aggregates are 2.6 and 2.4, respectively. The bulk specific gravity of the mix is 2.3.

The percentage air voids in the mix is ____________. (round off to the nearest integer)

Key: **(7.5 to 8.5)**

Sol: Given that,

Coarse aggregate = 260 gms

Fine aggregate = 240 gms

$G_{CA} = 2.6$

$G_{FA} = 2.4$

Bulk specific gravity $(G_m) = 2.3$

Percentage air voids in the mix = ?

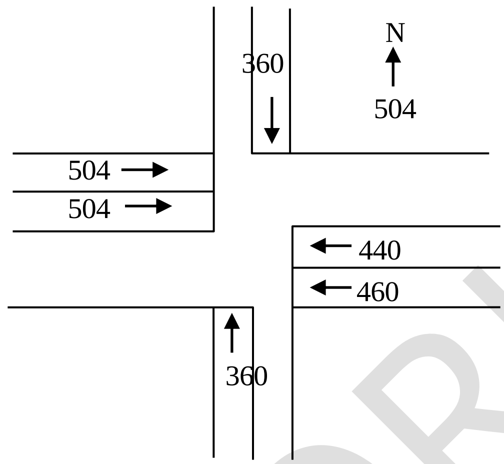
G_t (Theoretical specific gravity)

$$= \frac{\sum W}{\sum \frac{W}{G}} = \frac{260+240}{\frac{260}{2.6} + \frac{240}{2.4}} = 2.5$$

% air voids $(V_V) = \frac{G_t - G_m}{G_t} \times 100 = \frac{2.5-2.3}{2.5} \times 100 = 8\%$

$(V_V) = 8\%$

65. The lane configuration with lane volumes in vehicles per hour of a four-arm signalized intersection is shown in the figure. There are only two phases: the first phase is for the East-West and the West-East through movements, and the second phase is for the North-South and the South-North through movements. There are no turning movements. Assume that the saturation flow is 1800 vehicles per hour per lane for each lane and the total lost time for the first and the second phases together is 9 seconds.



The optimum cycle length (in seconds), as per the Webster's method, is ____________. (round off to the nearest integer)

Key: **36 to 38)**

Sol: $y_{N-S} = \left[\frac{360}{1800}, \frac{396}{1800}\right]_{max}$

$y_{N-S} = \frac{396}{1800} = 0.22$

$y_{E-W} = \left[\frac{504+504}{2\times 18.0}, \frac{40}{1800}, \frac{460}{1800}\right]_{max} = 0.28$

Optimum Cycle length

$= \frac{1.5L+5}{1-y}$

$= \frac{1.5\times 9+5}{1-(y_{N-S}+y_{E-W})}$

$= \frac{1.5\times 9+5}{1-(0.22+0.28)}$

$C_0 = 37\text{ sec}$